National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Assessment Resource

Mathematics and Statistics Level 2

Resource title: Waikato Cycleway

This exemplar supports assessment against:

Achievement Standard 91260

Apply network methods in solving problems

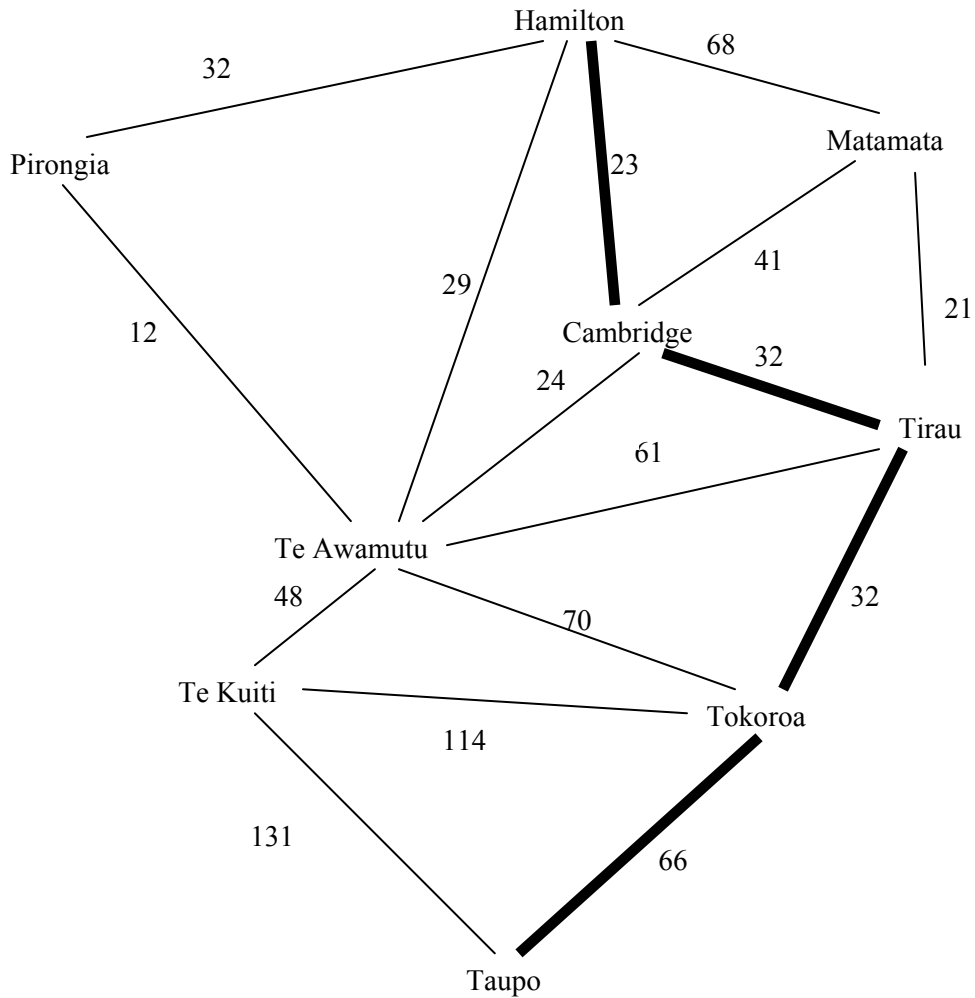Student and grade boundary specific exemplar

The material has been gathered from student material specific to an A or B assessment resource.

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This task involves students applying networks to design cycleways that meet given priorities.

	Grade Boundary: Low Excellence
1.	For Excellence, a student is required to apply networks, using extended abstract thinking in solving problems. This involves devising a strategy to investigate a situation, identifying relevant concepts in context and using correct mathematical statements or communicating mathematical insight. This student has devised a strategy to investigate the paths and has identified relevant concepts in context to develop a compromise network (3). For a more secure Excellence the minimum spanning tree for the minimum price needs to be correct (1) and the justification for the maximum spanning tree for the scenic value needs to be developed (2).

Hamilton

32

68

Pirongia

Matamata

23

41

21

12

29

Cambridge

32

24

Tirau

61

Te Awamutu

32

48

70

Te Kuiti

Tokoroa

114

131

66

Taupo

The shortest at the Taupo club's request is
Hamilton to Cambridge to Tirau to Tokoroa to Taupo
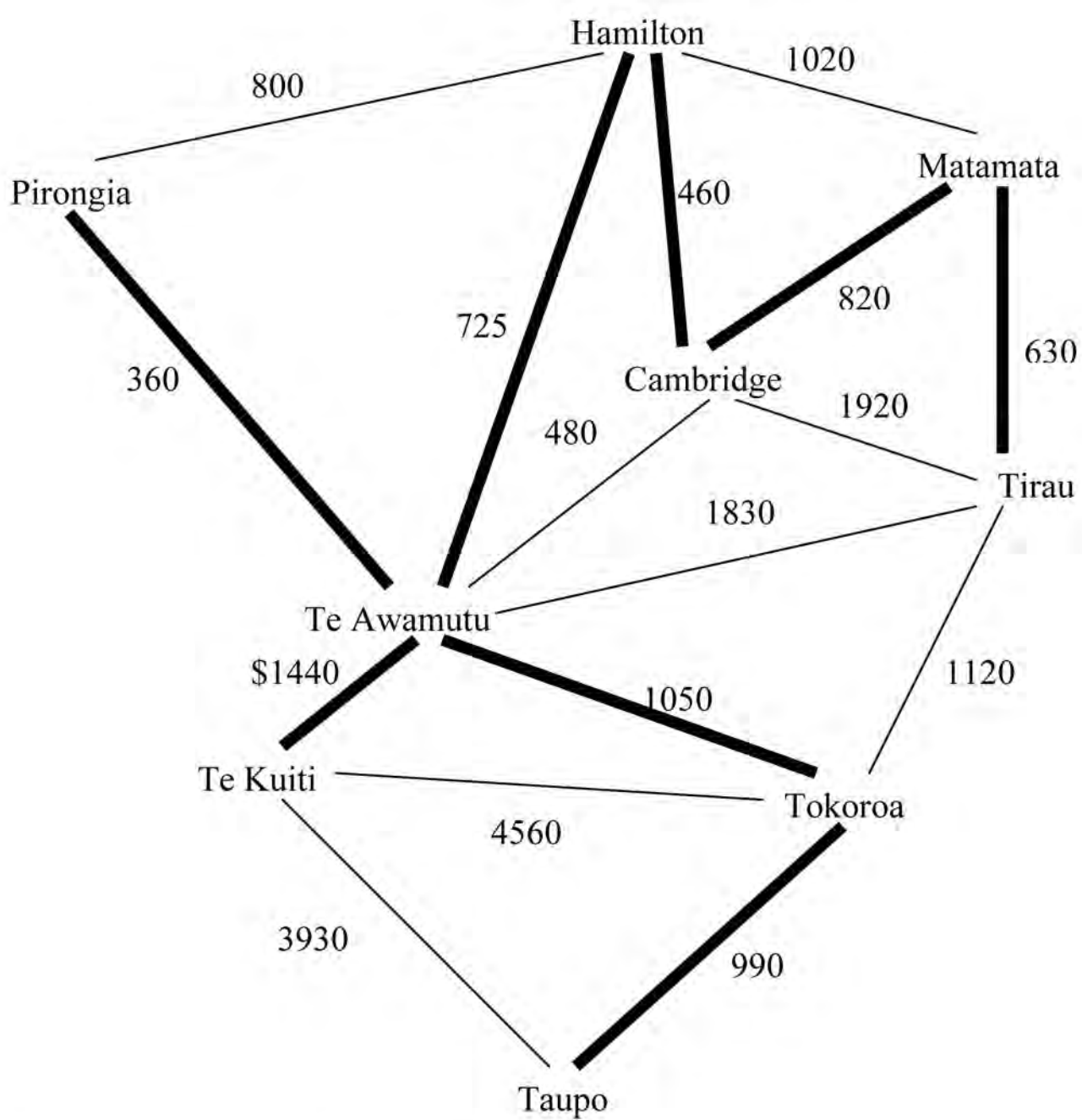23 + 32 + 32 + 66 = 153 km

I have chosen this route because all the other routes are further and the question asks
for the shortest possible route. For example if I had gone
Hamilton – Te Awamutu – Tokoroa – Taupo = 29 + 70 + 66 = 165 km which is
further than the path I have highlighted by 12 km.
Other routes are all longer
Hamilton – Matamata – Tirau – Tokoroa – Taupo = 68 + 21 + 32 + 66 = 187 km
Hamilton – Te Awamutu – Te Kuiti – Taupo = 29 + 48 + 131 = 208 km
Hamilton – Cambridge – Te Awamutu – Tokoroa – Taupo = 23 + 24 + 70 + 66 = 183
km

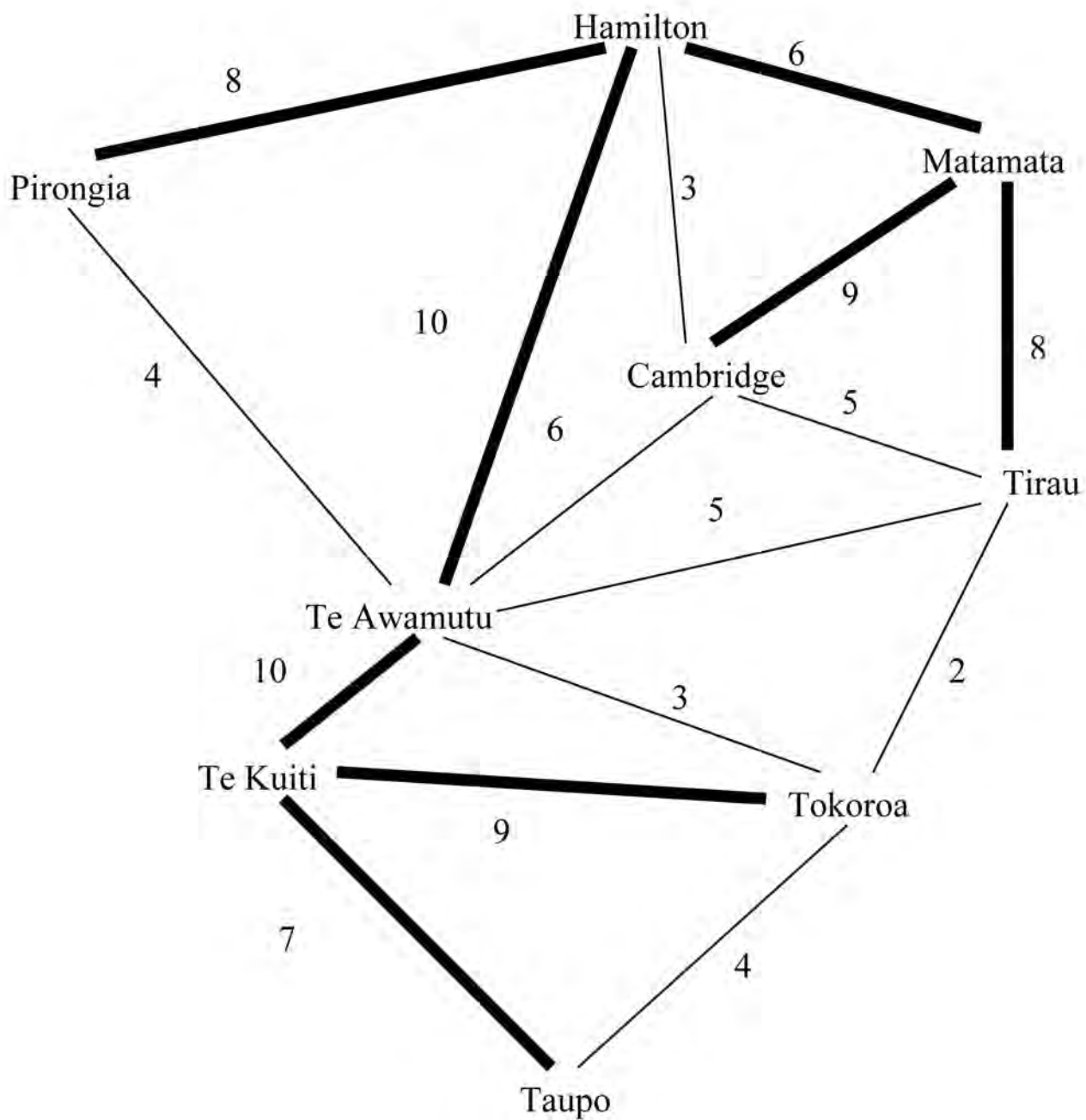Hamilton

800

Pirongia

1020

Matamata

460

360

725

Cambridge

820

630

480

1920

Tirau

1830

Te Awamutu

$1440

1050

1120

Te Kuiti

Tokoroa

4560

3930

990

Taupo

Hamilton club wants Minimum spanning tree.

360 + 460 + 630 + 725 + 820 + 990 + 1050 + 1440 = $6,475,000
Price $ 6,475,000   (1)

I have chosen this route because if I chose another path this would increase my total cost. For example if I went from Hamilton to Pirongia rather than Hamilton to Te Awamutu this would increase my price by $75,000.

Tirua Clubs's request

Hamilton → Pirongia (8)   Hamilton → Te Awamutu (10)
Matamata → Cambridge (9)   Matamata → Tirau (8)
Hamilton → Matamata (6)   Te Awamutu → Te Kuiti (10)
Te Kuiti → Tokoroa (9)   Te Kuiti → Taupo (7)

Maximum scenic value = 67

I have chosen this route because if I chose another path this would decrease my scenic value. Where there is a choice always select the highest one which is the highest scenic value. For example for Pirongia choose 8 to Hamilton and not 4 to Te Awamutu. (2)
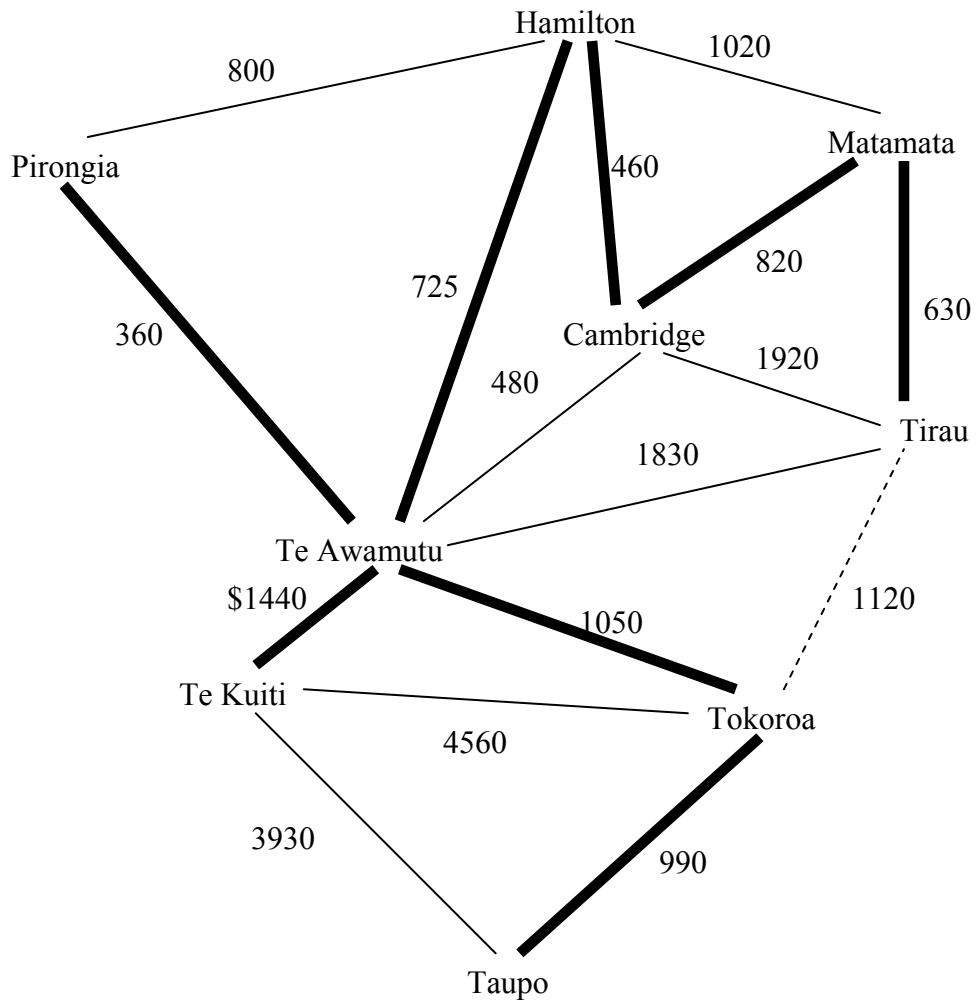
If I add an arc as shown above (dashed line) to cost of the cycleway from Tirau to Tokoroa this will provide a route Hamilton to Taupō which is close to the shortest path (it adds approximately 32 km). Having added one extra path will increase my cost by $1,120,000. It will also reduce the scenic value from 67 to 49.
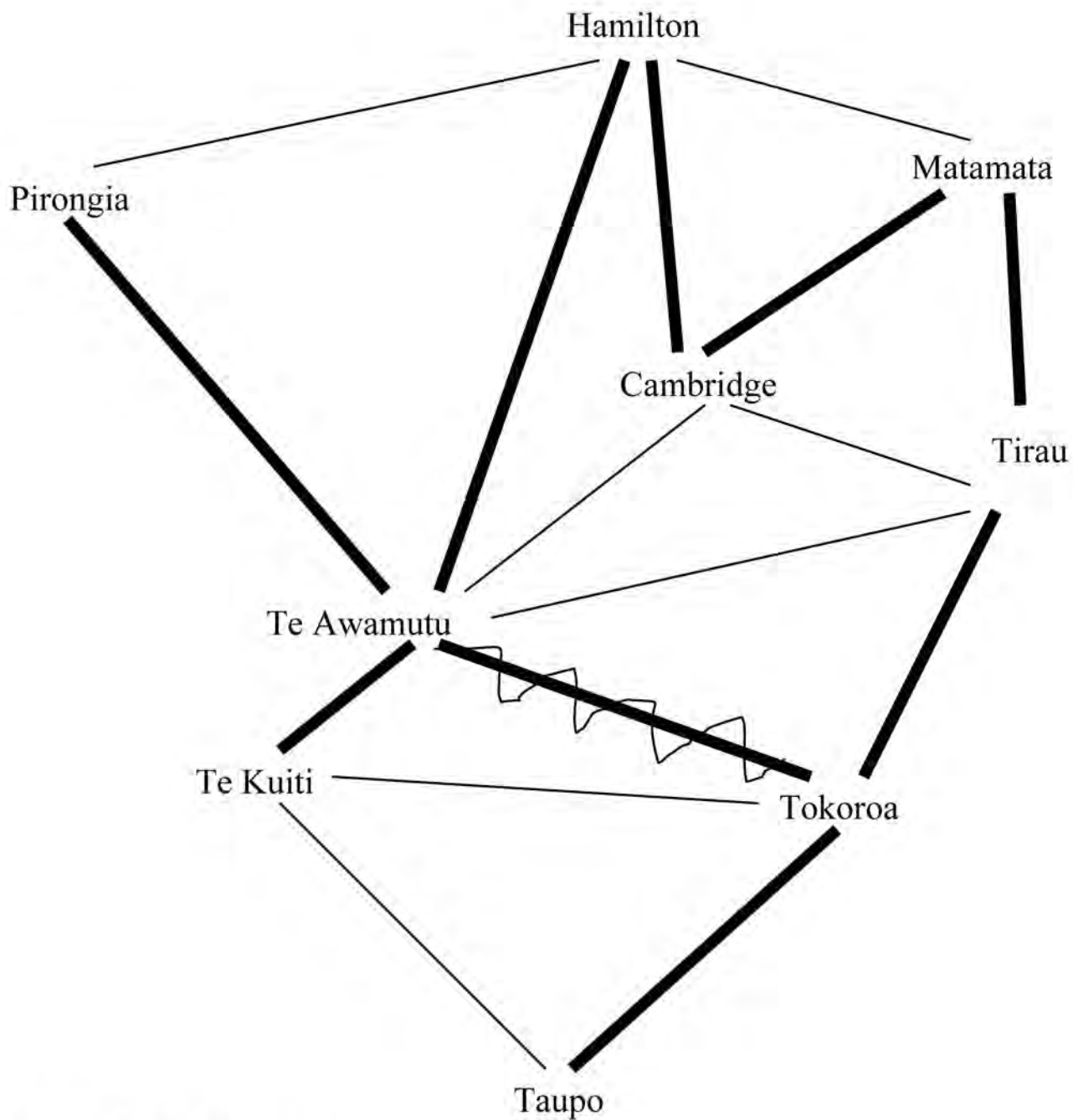
Tokoroa Club Request:

Is the path traversable?
No the path is not traversable because two of the nodes are odd. Because of these two odd nodes, (Matatmata and Te Kuiti), you can't start at Tokoroa and get back to Tokoroa.

Decided path: ③

Te Kuiti → Te Awamutu; (48 km, 10, $1440 (thousand))
Pirongia → Te Awamutu: (12 km, 4, $360 (thousand))
Te Awamutu → Hamilton: (29 km, 10, $725 (thousand))
Hamilton → Cambridge: (23 km, 3, $460 (thousand))
Cambridge → Matamata: (41 km, 4, $820 (thousand))
Matamata → Tirau: (21 km, 8, $630 (thousand))
Tirua → Tokoroa: (32 km, 2, $1120 (thousand))
Tokoroa → Taupo: (66 km, 4, $990 (thousand))
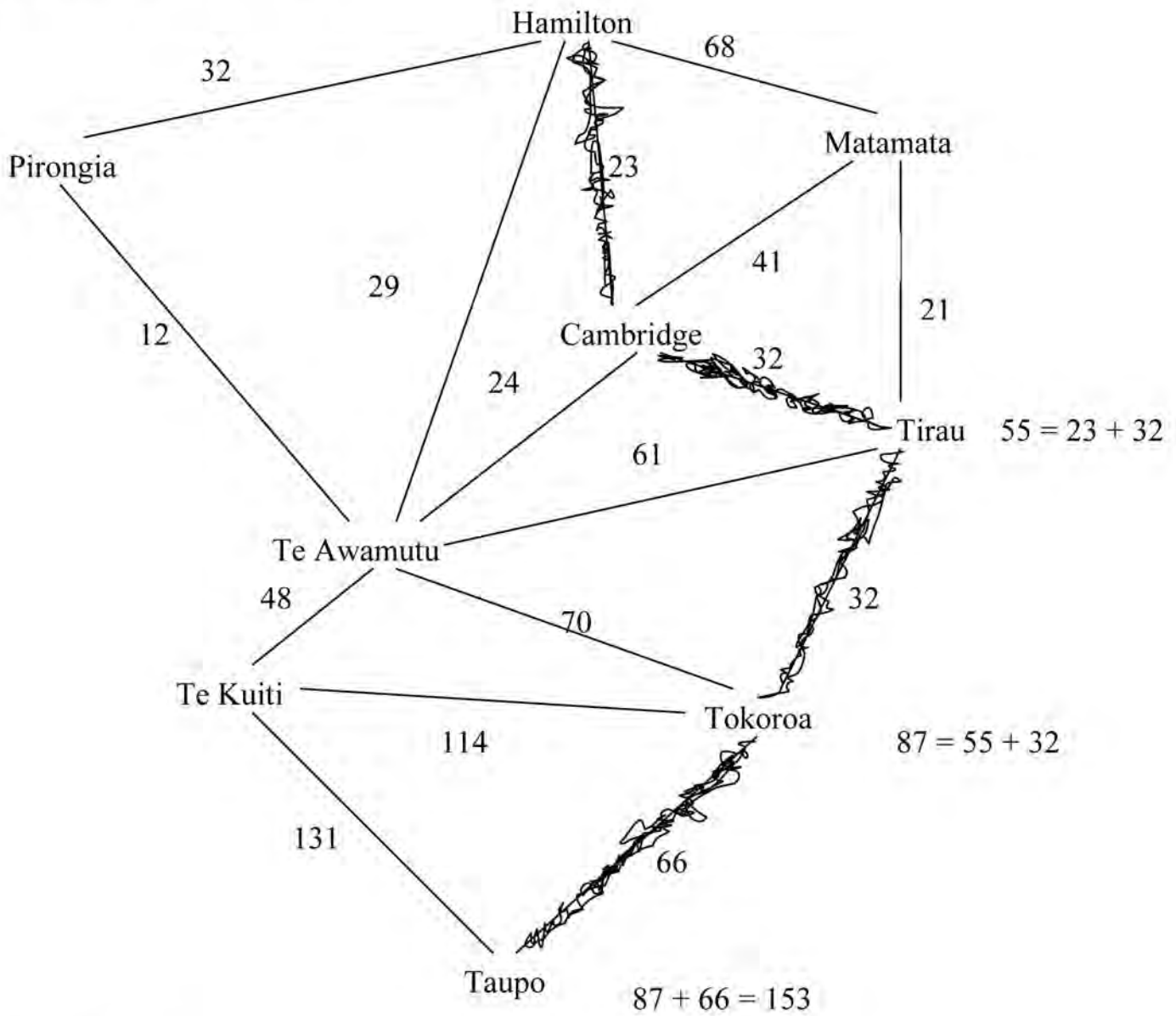
Overall km = 272
Overall Scenic score = 45
Overall money spent on construction$6,545,000

I believe this is the best route for everyone's needs, as I think it's balanced as to who benefits. Paths that were only beneficial to one club, that way majority rules. Unfortunately the Tokoroa request cannot be met i.e starting and finishing there.

	Grade Boundary: High Merit
2.	For Merit a student is required to apply networks, using relational thinking in solving problems. This involves demonstrating understanding of concepts and relating findings to a context or communicating thinking using appropriate mathematical statements. The justification for the shortest path, minimum spanning tree and traversability and the recognition of the alternative path in the maximum spanning tree provide evidence of an understanding of concepts (1), (2), (3), (4). To provide evidence of extended abstract thinking for excellence, consideration of a compromise network is required.

Taupo club – shortest route

Hamilton

32

Pirongia

68

Matamata

23

29

41

21

12

Cambridge

32

24

Tirau   55 = 23 + 32

61

Te Awamutu

32

48

70

Te Kuiti

Tokoroa   87 = 55 + 32

114

131

66

Taupo   87 + 66 = 153

The Shortest route is.
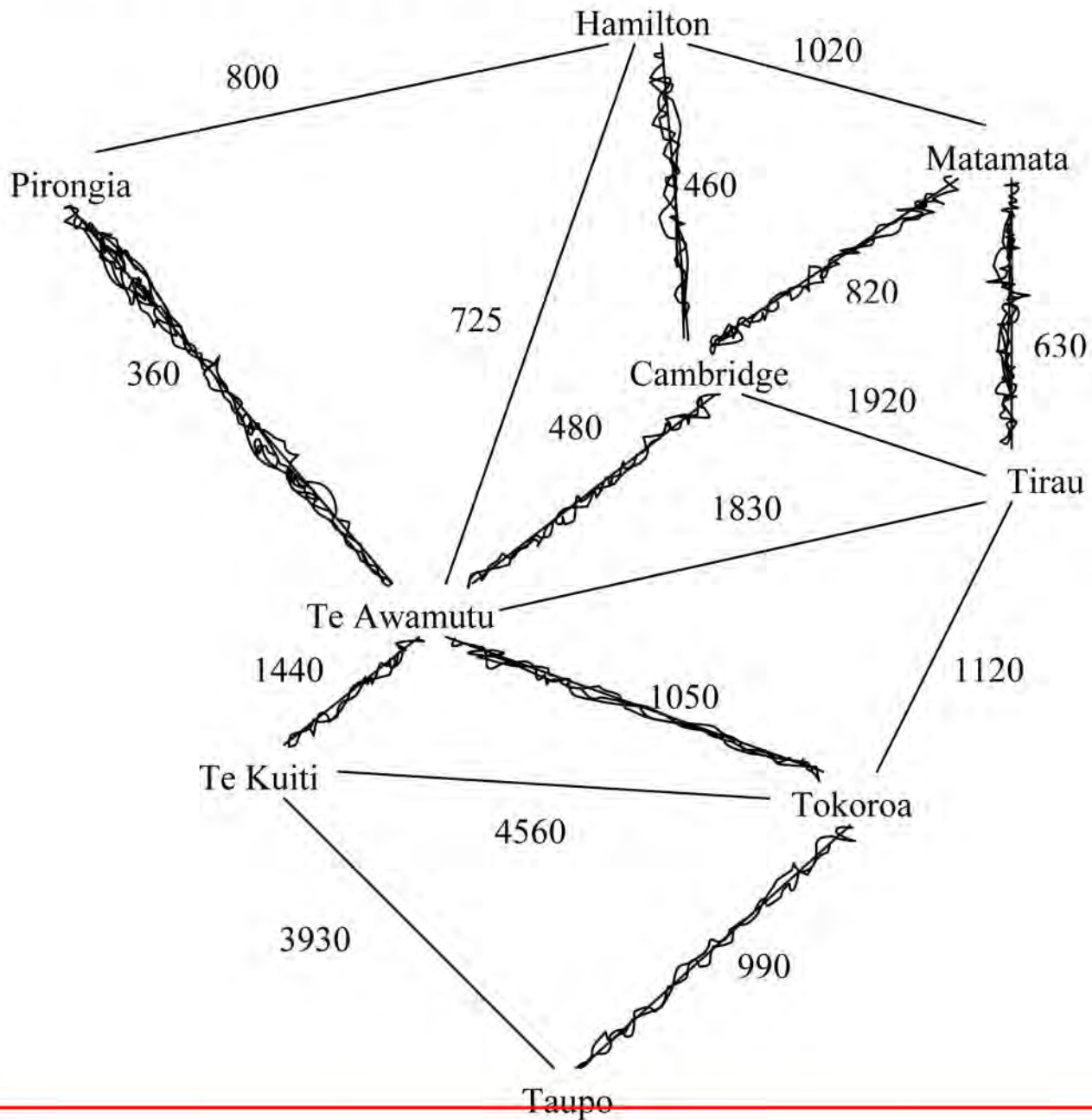
H → C → Tir → Tok → Taupo

Total distance = 153 Km

① I have chosen this route because all the other routes are further.
For example to get to Tir another way is H → M → Tir 68 + 21 = 89 so go H → C → Tir
To get to Tok was 87. To go H → Tea → Tok 29 + 70 = 99 is further.
There is no shorter way to get to Tea. H → C → Tea 23 + 24 = 47 H → P → Tea 32 + 12 = 44 but H → Tea only 29

Hamilton Club: Minimum cost (unit $1000)

Hamilton

800

1020

Matamata

Pirongia

460

820

630

725

360

Cambridge

1920

480

Tirau

1830

Te Awamutu

1440

1120

1050

Te Kuiti

Tokoroa

4560

3930

990

Taupo

②

There are 9 cities. I will select 8 paths for the minimum spanning tree. (choose from cheapest to the expensive and make sure all cities are connected 360 + 460 + $480 + 630 + 820 + 990 + 1050 + 1440 = $6,230,000)

Always select the cheapest path for a town. To get to Pirongia use 360, not 800 and for Taupo use 990 not 3630. Tirau has to be 630, not 1920, 1830 or 1120 which are more.
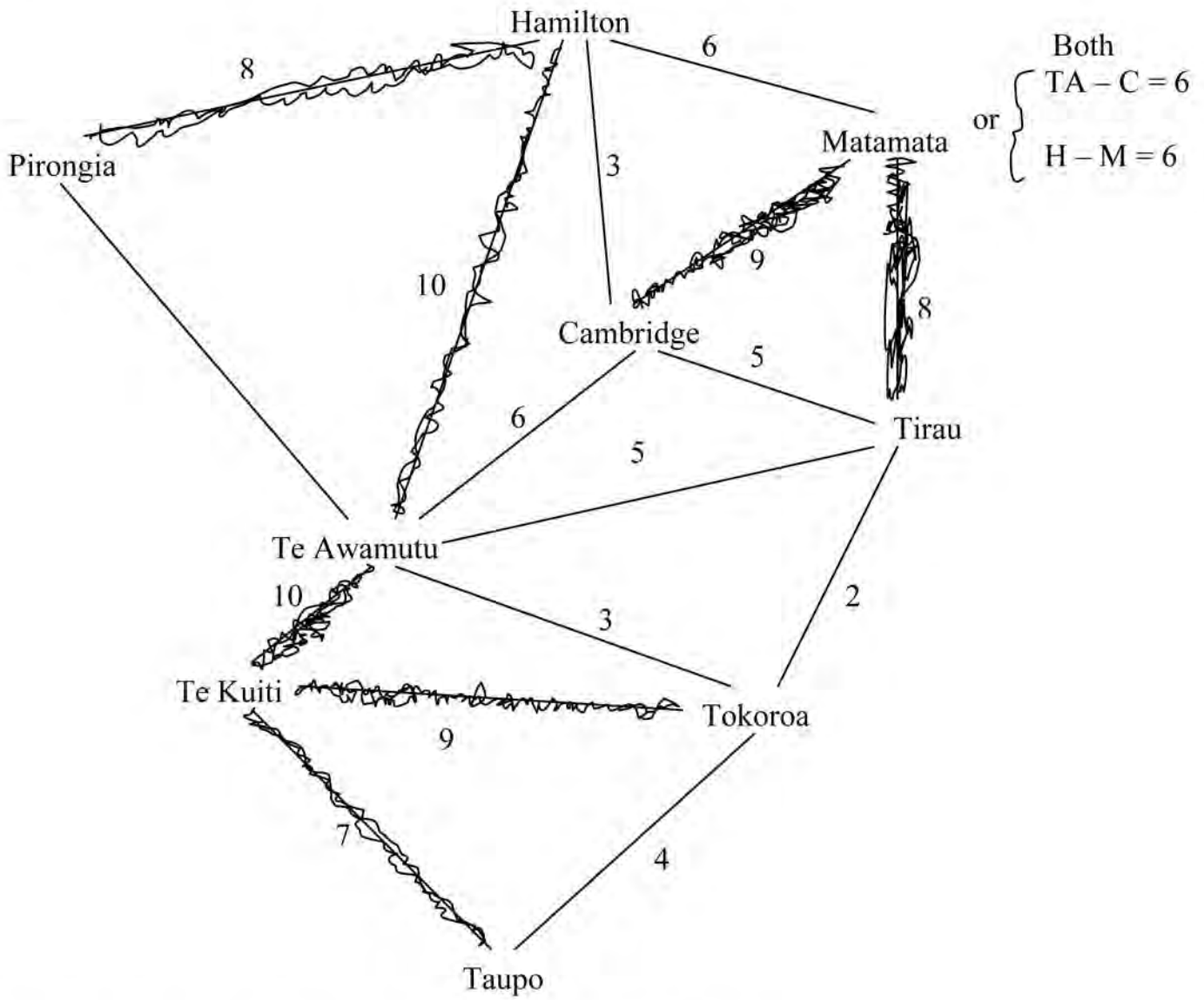
I have chosen this route because if I chose another path this would increase my total cost. For example if I had gone Hamilton to Te Awamutu rather than Cambridge to Te Awamutu

360 + 460 + 630 + 725 + 820 + 990 + 1050 + 1440 = $6,475,000

Price $ 6,475,000

This would increase the cost by $245,000

Tirau Club – Maximum scenic values

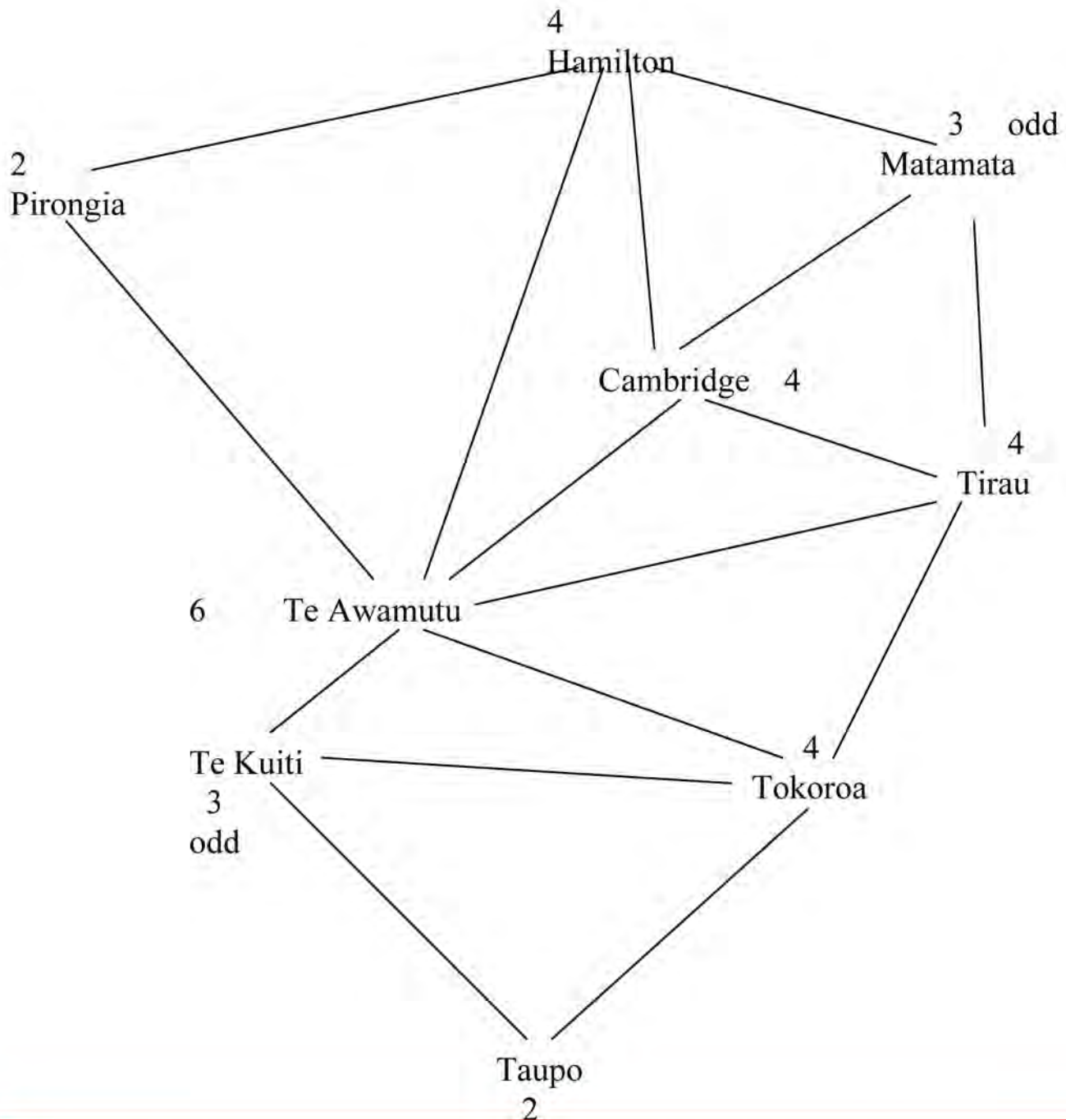Choose 8 paths with no loop. Chose the largest scenic values to the lowest.
10 + 10 + 9 + 9 + 8 + 8 + 7 + 6) = 67

Either TA – C or H - M ④

Scenic value = 67

Tirua Clubs request
Maximum scenic value = 67

4
Hamilton

3   odd
Matamata

2
Pirongia

Cambridge   4

4
Tirau

6   Te Awamutu
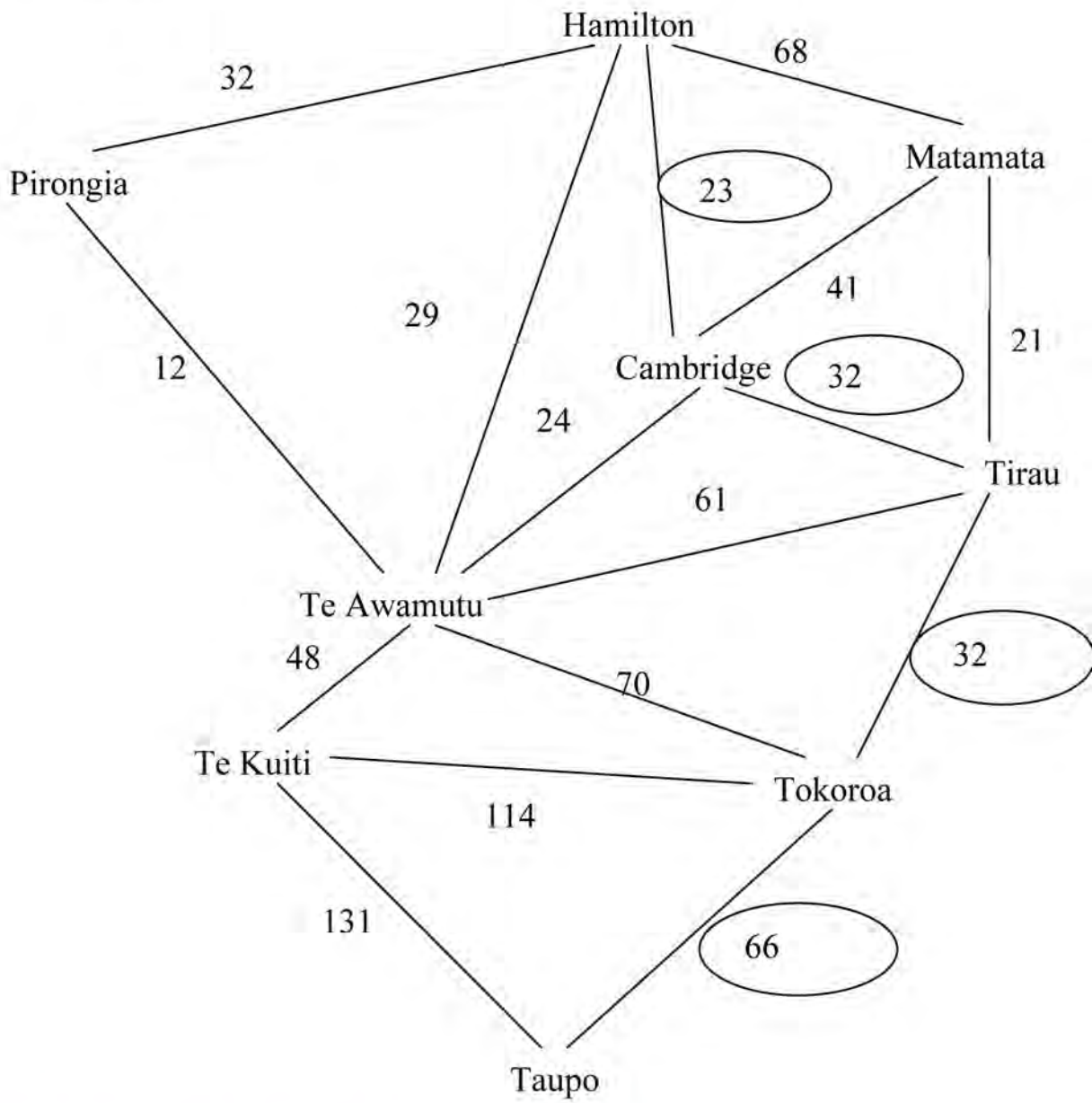
Te Kuiti
3
odd

4
Tokoroa

Taupo
2

③

The network is not traversable from Tokoroa. To be traversable the network must have all even nodes or exactly 2 odd nodes. This network has two odd nodes (Te Kuiti and Matamata). In the event of any odd nodes you cannot start and finish at the same point.

|	Grade Boundary: Low Merit
3.	This student has demonstrated an understanding of concepts in finding the shortest path by exhaustion (1), the explanation for traversability and investigation of a path that would start and end in Tokoroa (2). While the networks for minimum and maximum spanning trees are correct, there is a misunderstanding of the function of the maximum spanning tree for the scenic value and no justification for the paths that have been selected (3). For a more secure Merit there needs to be a justification for the paths that have been selected for the minimum and maximum spanning trees, and an accurate understanding of the scenic value in the maximum spanning tree.

Dist (Taupo Club)

Hamilton

32

68

Pirongia

23

Matamata

29

41

21

12

Cambridge

32

24

61

Tirau

Te Awamutu

32

48

70

32

Te Kuiti

Tokoroa

114

131

66

Taupo

Shortest route Hamilton - Taupo.

Ham – Cam – Tir – Tok – Tau: = 153 Km (1)

Ham – Tea – Toko – Tau: = 165 Km

Ham – Tea – Tek – Tau: = 208 Km

Ham – Cam – Tea – Tok – Tau = 183 km

Shortest Ham – Cam – Tir – Tok – Tau: = 153 Km

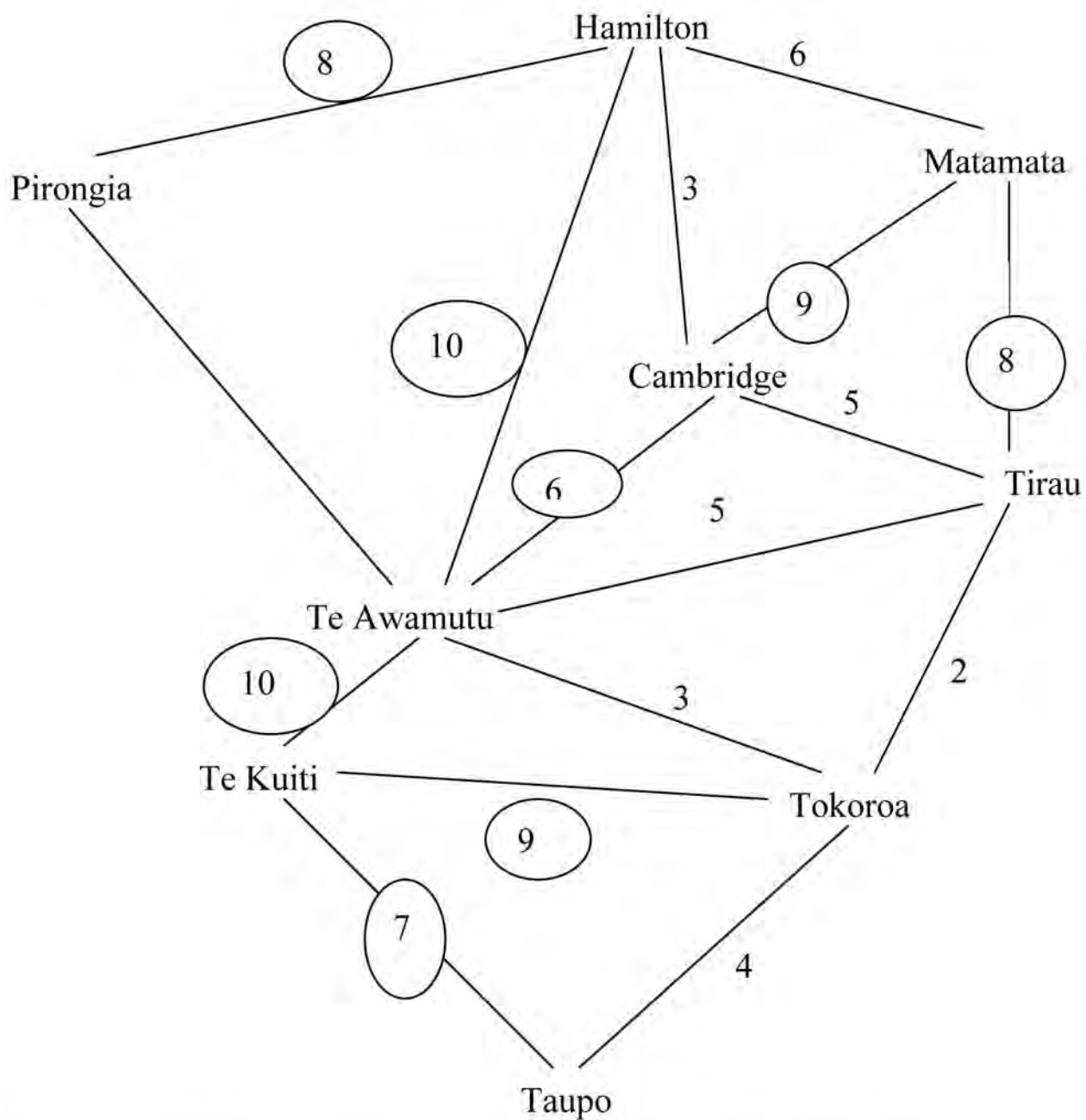$$ (Hamilton
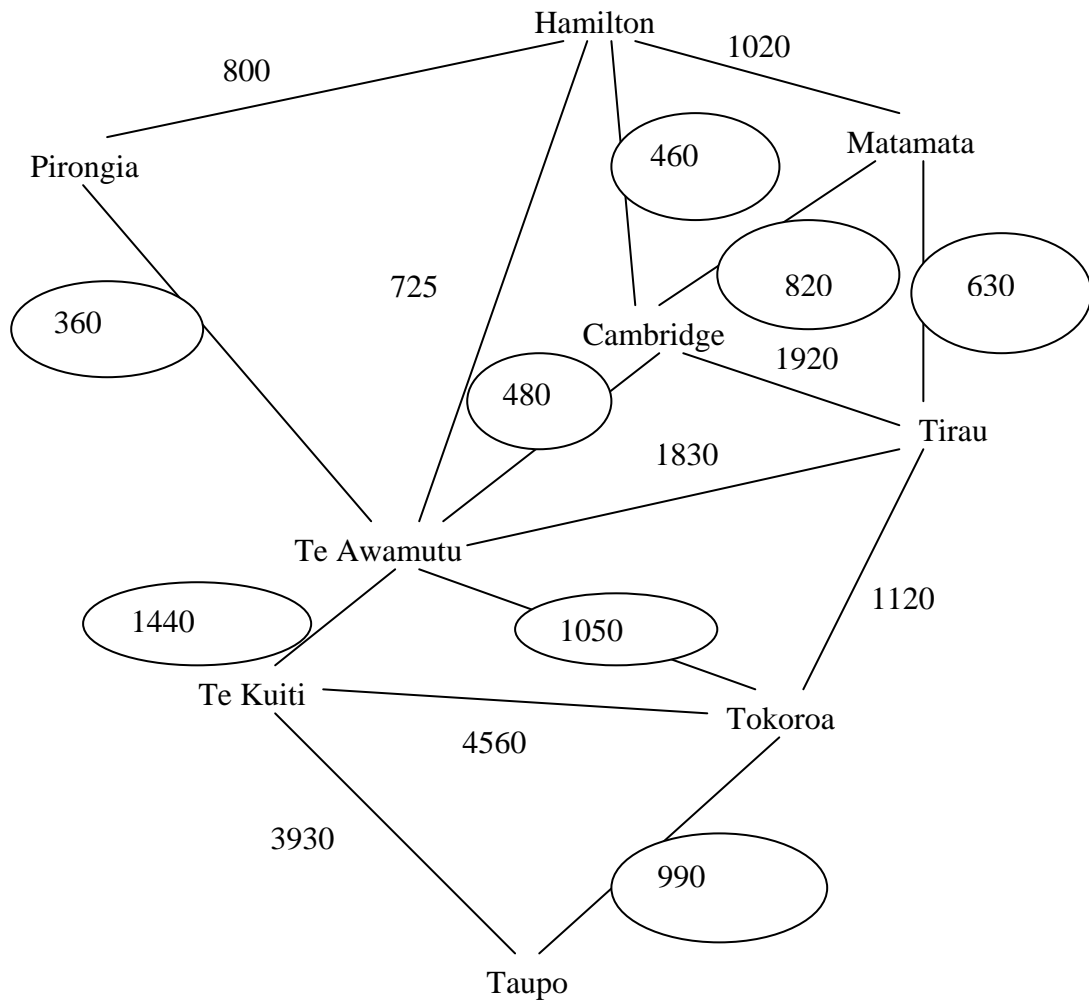
Hamilton club wants Minimum spanning tree.

460 + 480 + 820 + 630 + 1440 + 1050 + 990 + 360 = 6,230,000

Scenery (Tirau)

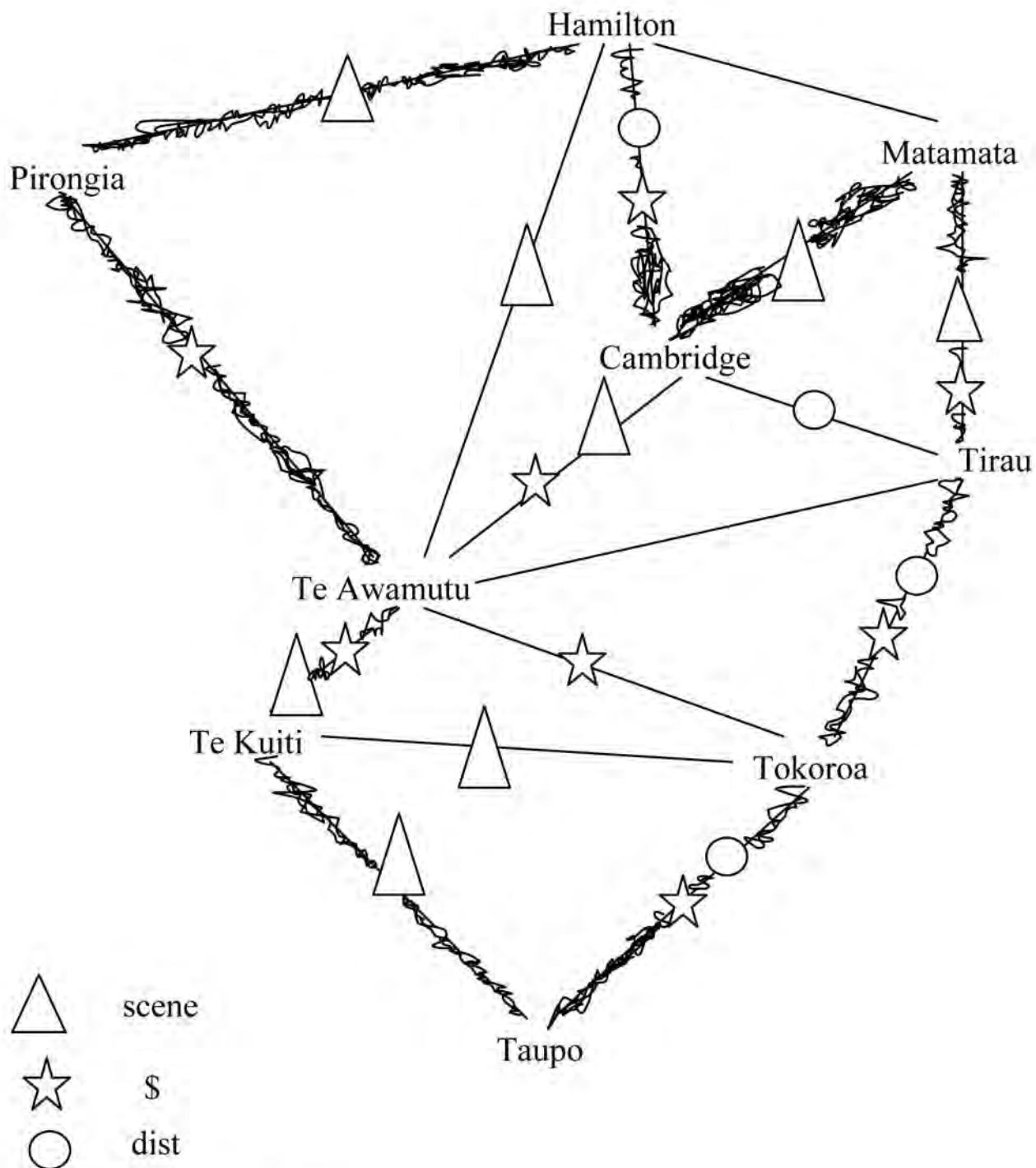Highest scenic value would be measurable by mean since total is silly given you could keep adding paths. ③

Maximum spanning three is circled. (10 + 10 + 9 + 9 + 8 + 8 + 7 +6) /8 = 8.375

Traversability (Tokoroa)

The network is not traversable. To be traversable you would need to start at either of these 2 cities (Te Kuiti and Matamata) however you would still not be able to get back to your starting point. ②

So what Tokoroa want isn't possible – the network can't be traversed from Tokoroa since 2 towns are of odd nodes (Te Kuiti and Matatmata) so start/stop can't be the same place.
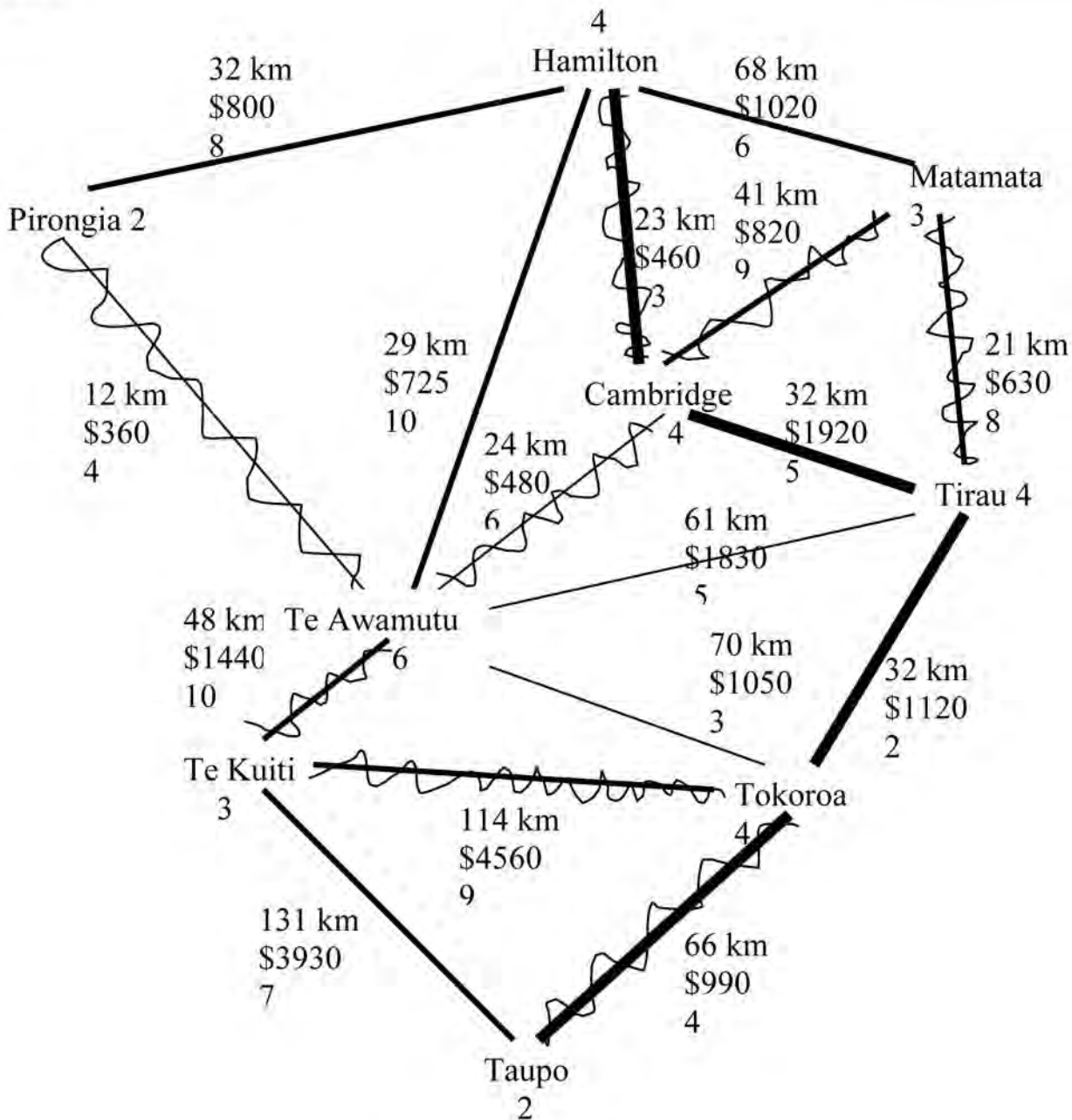
Proposed

Scene △

$ ☆

dist ○

Dist Ham – Taupo 183 km
$ 10,550,000
Scene 6.11

	Grade Boundary: High Achieved
4.	For Achieved a student is required to apply networks, in solving problems. This involves selecting and using methods associated with networks, demonstrating knowledge of concepts and terms associated with networks and communicating using appropriate representations. The paths for the minimum and maximum spanning trees have been stated and the student has demonstrated some understanding of traversability. The student has provided a justification for the network for the shortest path (1), (2), (3). For evidence of relational thinking for Merit, the student would need to relate their findings more clearly to the context with the use of appropriate units, demonstrate a clearer understanding of traversability and provide a justification of the minimum spanning trees.

Route 1

4
Hamilton

32 km
$800
8

68 km
$1020
6

Pirongia 2

Matamata

41 km
$820
9

23 km
$460
3

3

29 km
$725
10

21 km
$630
8

12 km
$360
4

Cambridge

32 km
$1920
5

4

24 km
$480
6

61 km
$1830
5

Tirau 4

48 km   Te Awamutu
$1440   6
10

70 km
$1050
3

32 km
$1120
2

Te Kuiti
3

Tokoroa
4

114 km
$4560
9

131 km
$3930
7

66 km
$990
4

Taupo
2

Hamilton – Cambridge – Tirau – Tokoroa – Taupo = 23 + 32 + 32 + 66 = 153

Hamilton – Te Awamutu – Te Kuiti – Taupo = 29 + 48 + 131 = 208

Hamilton – Matamata – Tirau – Tokoroa – Taupo = 68 + 21 + 32 + 66 = 187

Hamilton – Te Awamutu – Tokoroa – Taupo = 29 + 70 + 66 = 165

Hamilton – Cambridge – Te Awamutu – Tokoroa – Taupo = 23 + 24 + 70 + 66 = 183 (1)

I will choose Hamilton to Cambridge to Tirau to Tokoroa to Taupo because it is the shortest one.

Route 2

I will keep to the line (Hamilton – Cambridge, Matamata – Cambridge, Matamata – Tirau, Te Awamutu – Cambridge, Pirongia - Te Awamutu, Te Awamutu – Tokoroa, Te Awamutu – Te Kuiti, Tokoroa – Taupo)

Because it is the cheap one.

360 + 480 + 1050 + 460 + 820 + 630 + 990 + 1440 = 6230

Route 3 (2)

I will choose (Pirongia – Hamilton, Hamilton – Te Awamutu, Te Awamutu – Te Kuiti, Hamilton - Matatmata, Cambridge – Matamata, Te kuiti – Tokoroa, Te Kuiti – Taupo, Matamata – Tirau.
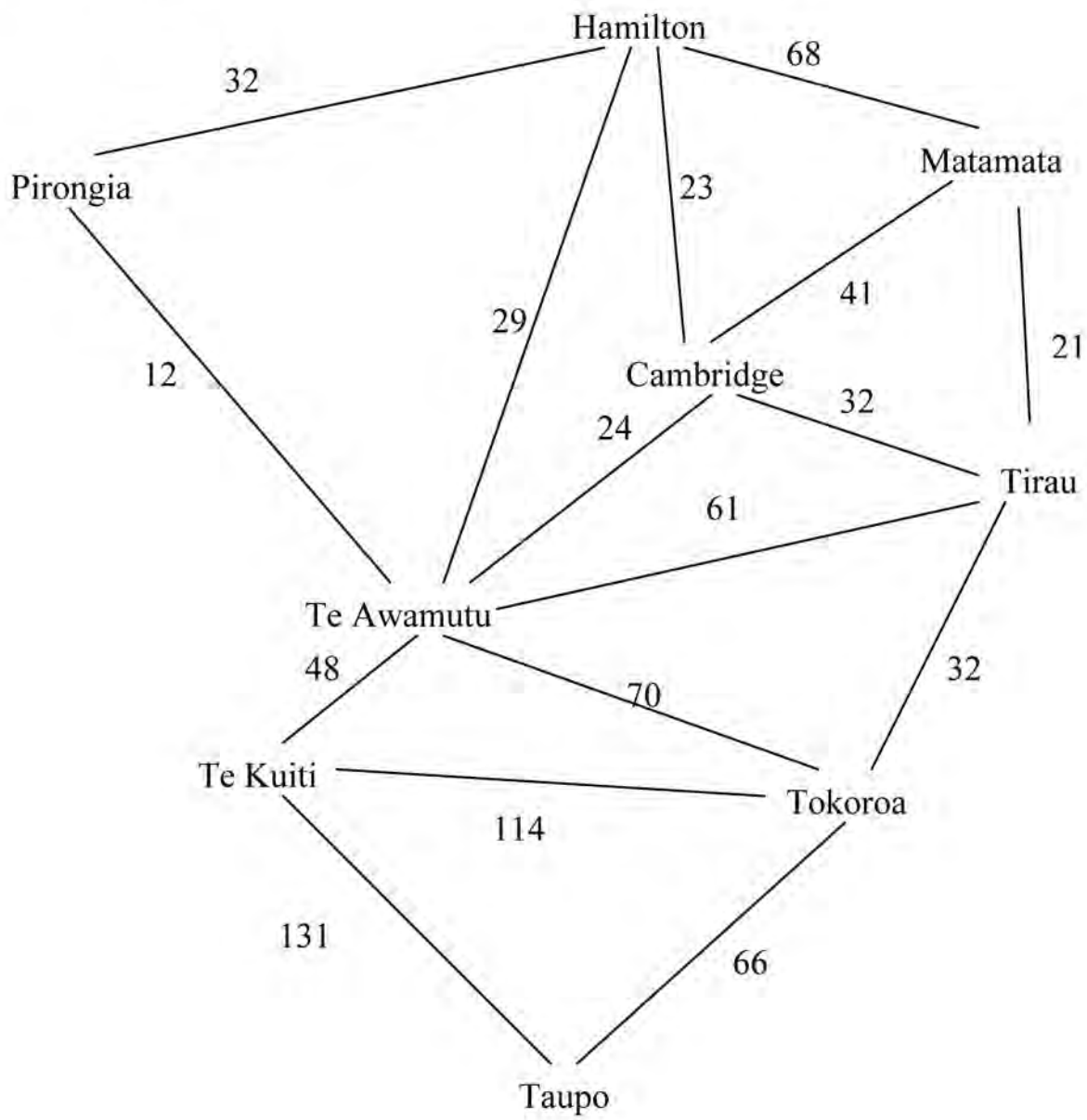
Because this line is the most popular people go to so the scenic value is = 67.

Route 4

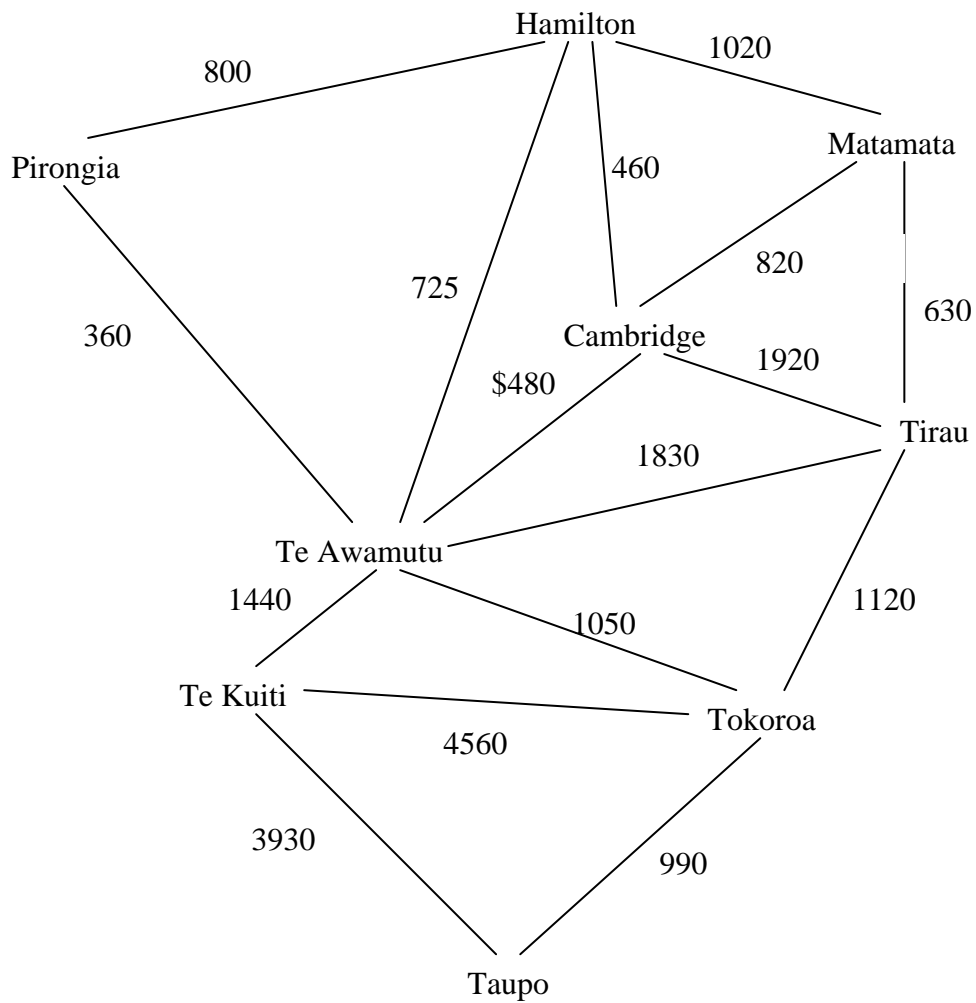This network has 2 odd nodes. It is not traversability as cannot get back to the starting node. (3)

	Grade Boundary: Low Achieved
5.	This student has selected and used methods associated with networks. The student has found the shortest path and correctly listed the paths needed for the maximum spanning tree (1), (2). For a more secure Achieved the student would need to correctly select the paths for the minimum spanning tree and get a correct total for the scenic value.

The Taupo Club



Hamilton – Cambridge – Tirau – Tokoroa - Taupo = 153 km ①

The Hamilton Club

Hamilton

800

Pirongia

1020

Matamata

460

725

820

Cambridge

630

360

$480

1920

Tirau

1830

Te Awamutu

1440

1050

1120

Te Kuiti

Tokoroa

4560

3930

990

Taupo

Minimum price to connect all towns = $7350

Hamilton → Cambridge,   Matamata → Cambridge,

Cambridge → Te Awamutu,   Te Awamutu → Pirongia,

Te Awamutu → Te Kuiti,   Tirau → Tokoroa,
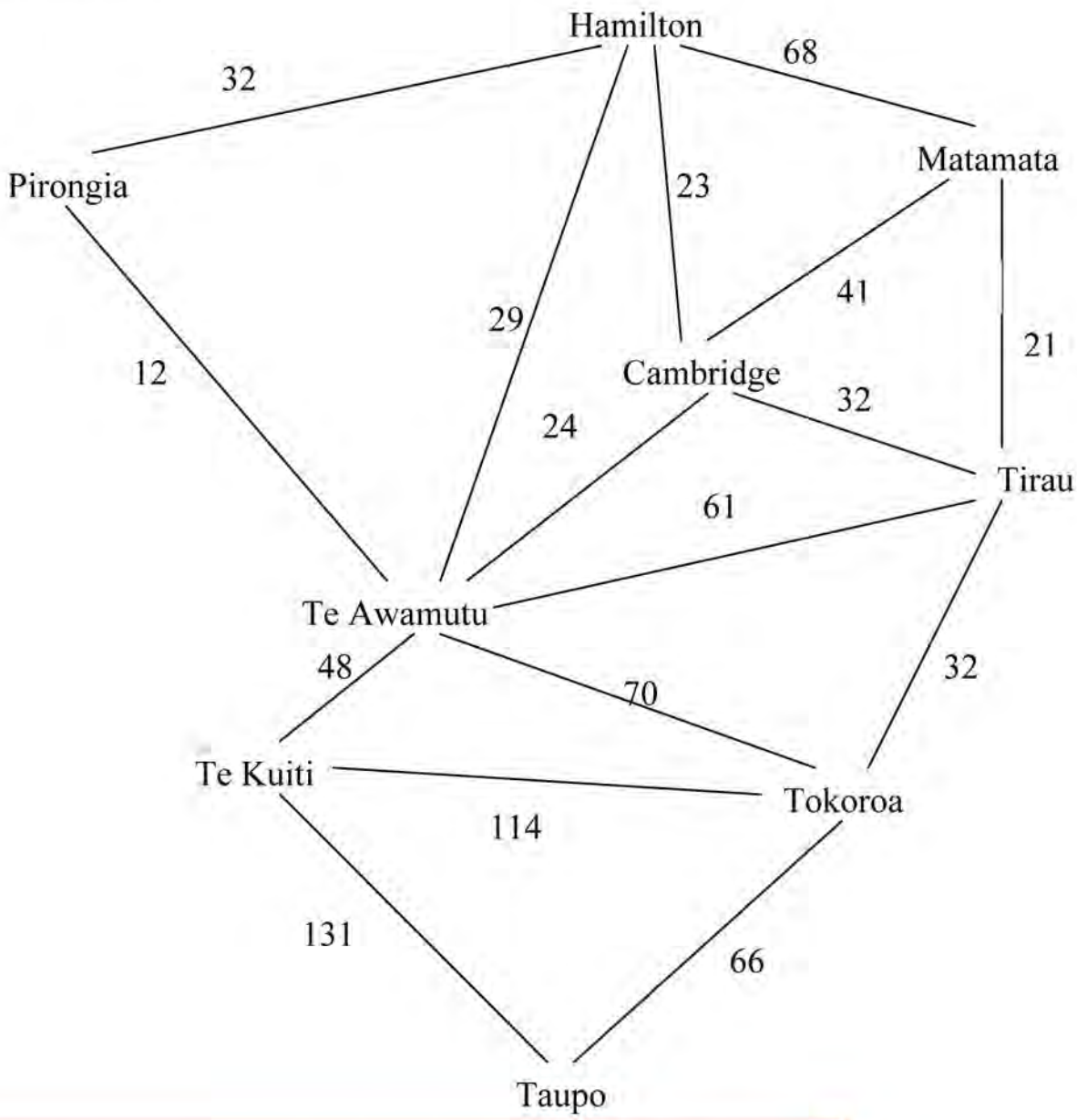
Te Awamutu → Tokoroa,   Tokoroa → Taupo

The Tirau Club
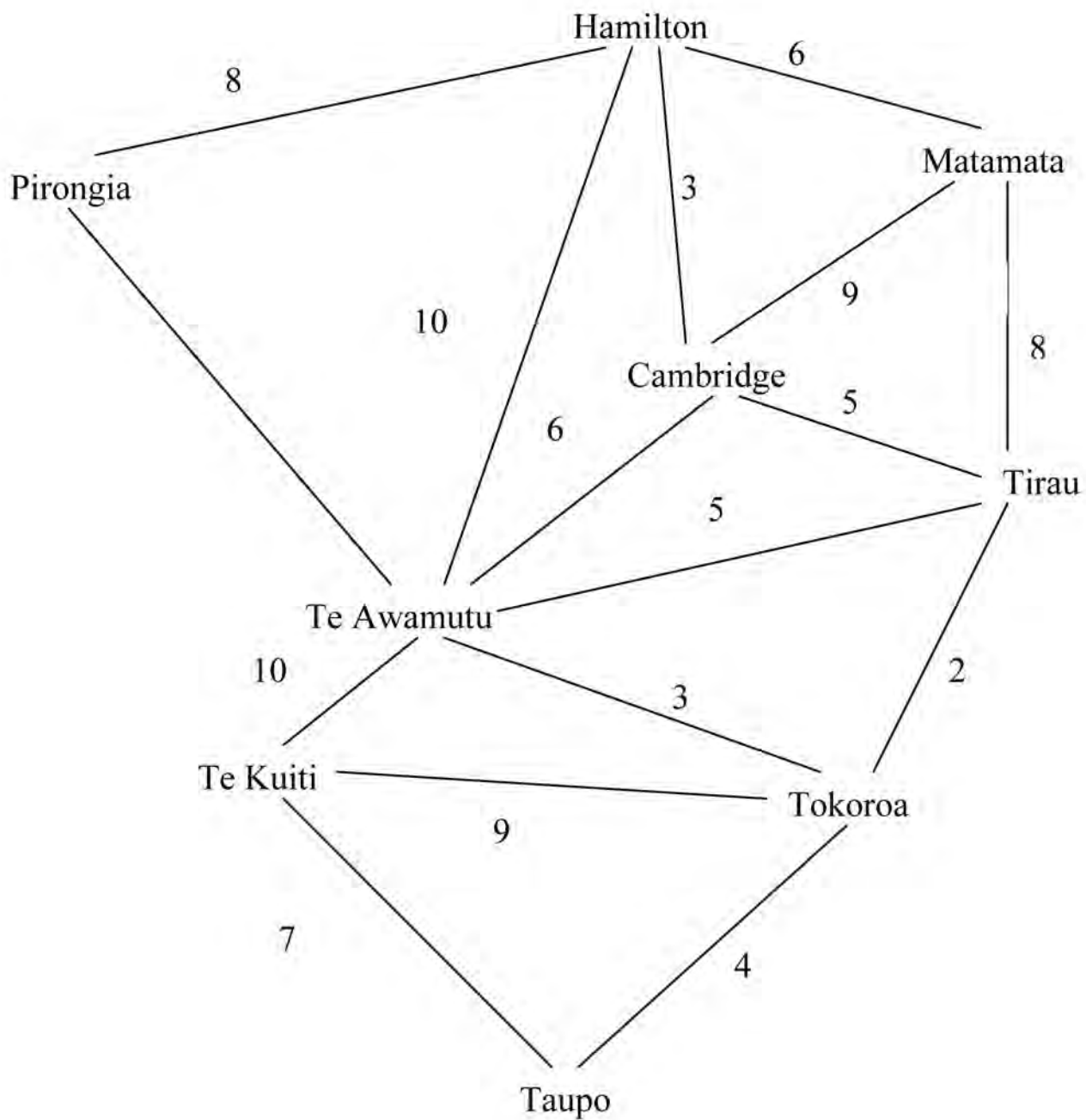
Maximum Scenic Route connecting all networks = 57

(2) Hamilton ⟶ Pironga,   Hamilton ⟶ Te Awamutu,

Matamata ⟶ Cambridge,   Matamata ⟶ Tirau,

Cambridge ⟶ Te Awamutu   Te Awamutu ⟶ Te Kuiti,

Te Kuiti ⟶ Tokoroa,   Te kuiti ⟶ Taupo

	Grade Boundary: High Not Achieved
6.	The student has found the shortest path between Hamilton and Taupo (1). There are some errors in the minimum spanning tree for the minimum price and a missing branch in the network for the maximum scenic value (2), (3). For the award of Achieved the student needs to correctly use at least two methods associated with networks.

Shortest route

Hamilton

32

68

Pirongia

Matamata

23

12

29

41

21

Cambridge

24

32

61

Tirau
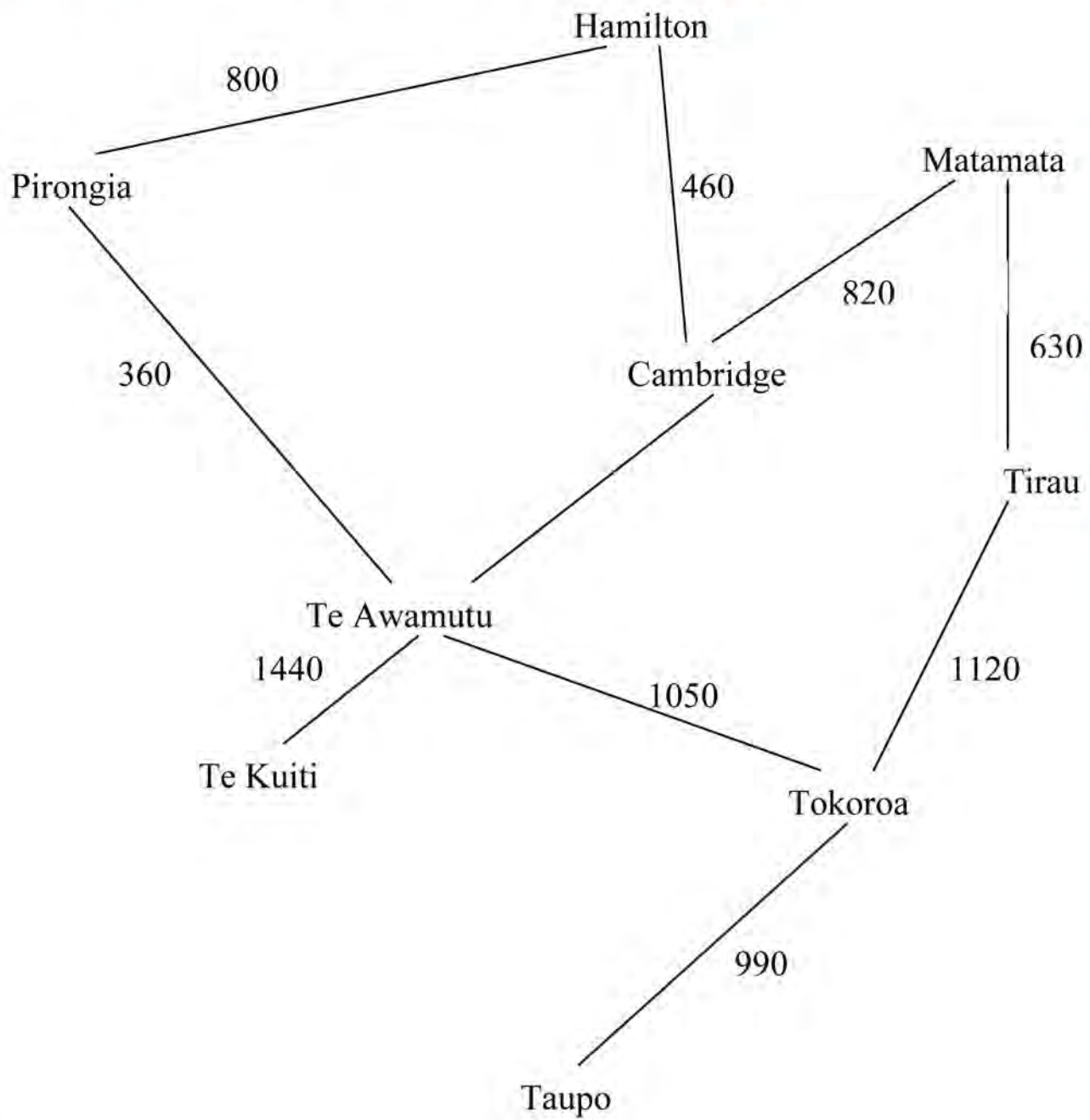
Te Awamutu

48

32

70

Te Kuiti

Tokoroa

114

131

66

Taupo

Shortest route between Hamilton and Taupo. = 153 Km ①

Hamilton to Cambridge to Tirua to Tokoroa to Taupo

Lowest cost to connect all towns

Hamilton

800

Pirongia

Matamata

460

360

820

630

Cambridge

Tirau

Te Awamutu

1440

1050

1120

Te Kuiti

Tokoroa

990

Taupo

Lowest Cost to connect all towns = $5612

Scenic value ③

Hamilton

8

Pirongia

6

Matamata

3

10

9

8

Cambridge

5

Tirau

6

5

Te Awamutu

5

10

2

3

Te Kuiti

Tokoroa

9

7

4

Taupo