National Certificate of Educational Achievement
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Exemplar for Internal Assessment Resource

Mathematics Level 3

Resource title: Machine Parts

This exemplar supports assessment against:

Achievement Standard 91574

Apply linear programming methods in solving problems

Student and grade boundary specific exemplar The material has been gathered from student material specific to an A or B assessment resource.

Date version published by Ministry of Education	December 2012 To support internal assessment from 2013

The task asks students to use the constraints provided to recommend the number of rods and pillars that the Ace Machine Company should produce to maximize the profit without increasing hours and how changing the number of drilling and grinding hours would affect this.

	Grade Boundary: Low Excellence
1.	For Excellence the student is required to apply linear programming methods, using extended abstract thinking, in solving problems. This involves devising a strategy to investigate or solve a problem, developing a chain of logical reasoning, and using correct mathematical statements or communicating mathematical insight. The student has shown evidence of extended and abstract thinking by providing the feasible region to satisfy all the constraints **(1)**, and has made a recommendation regarding the number rods and pillars to produce to maximise the profit **(2)**. The constraints and the feasible region have been produced for the increased number of hours **(3)**. The student has identified a theoretical optimal solution, and has provided a solution which recognises that the number of rods and pillars to produce must be integers **(4)**. For a more secure Excellence the student would need to accurately communicate their thinking relating to the whole number requirement for rods and pillars.

Student 1 page 1: Low Excellence

Rod = x
Pillar = y

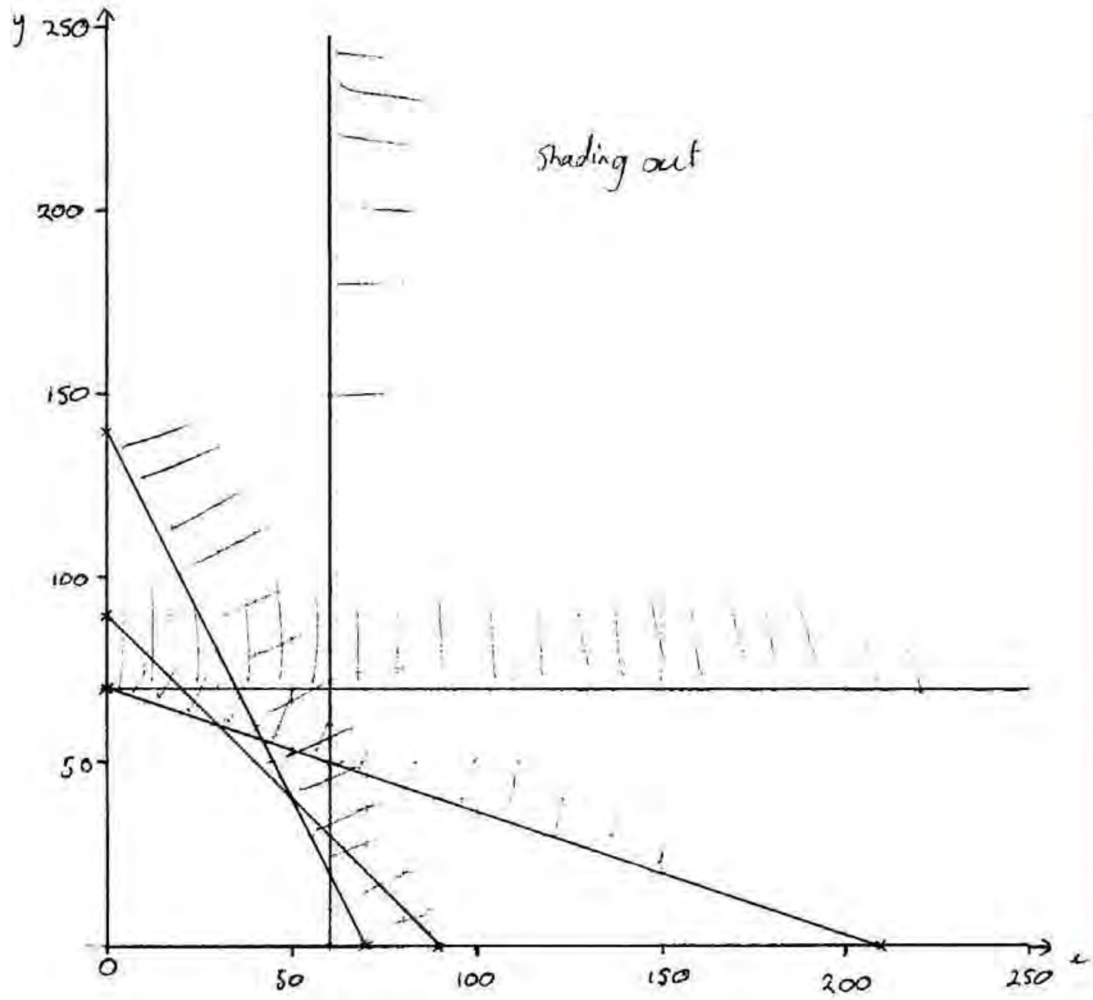
$x \le 60$
$y \le 70$

$0.5x + 1.5y \le 105$ $\quad y \le -\frac{1}{3}x + 70$ (drilling)

$x + y \le 90$ $\quad y \le -x + 90$ (grinding)

$x + 0.5y \le 70$ $\quad y \le -2x + 140$ (polishing)

$x \ge 0$
$y \ge 0$



1

$P = 300x + 600y$	(0, 70)	(30, 60)	(50, 40)	(60, 20)	(60, 0)
	42000	45000	39000	30000	18000

Max profit with current hours = \$45000
when rods made and sold = 30
and pillars made and sold = 60

2

new hours drilling $0.5x + 1.5y \le 110$ $y \le -\frac{1}{3}x + 73\frac{1}{3}$

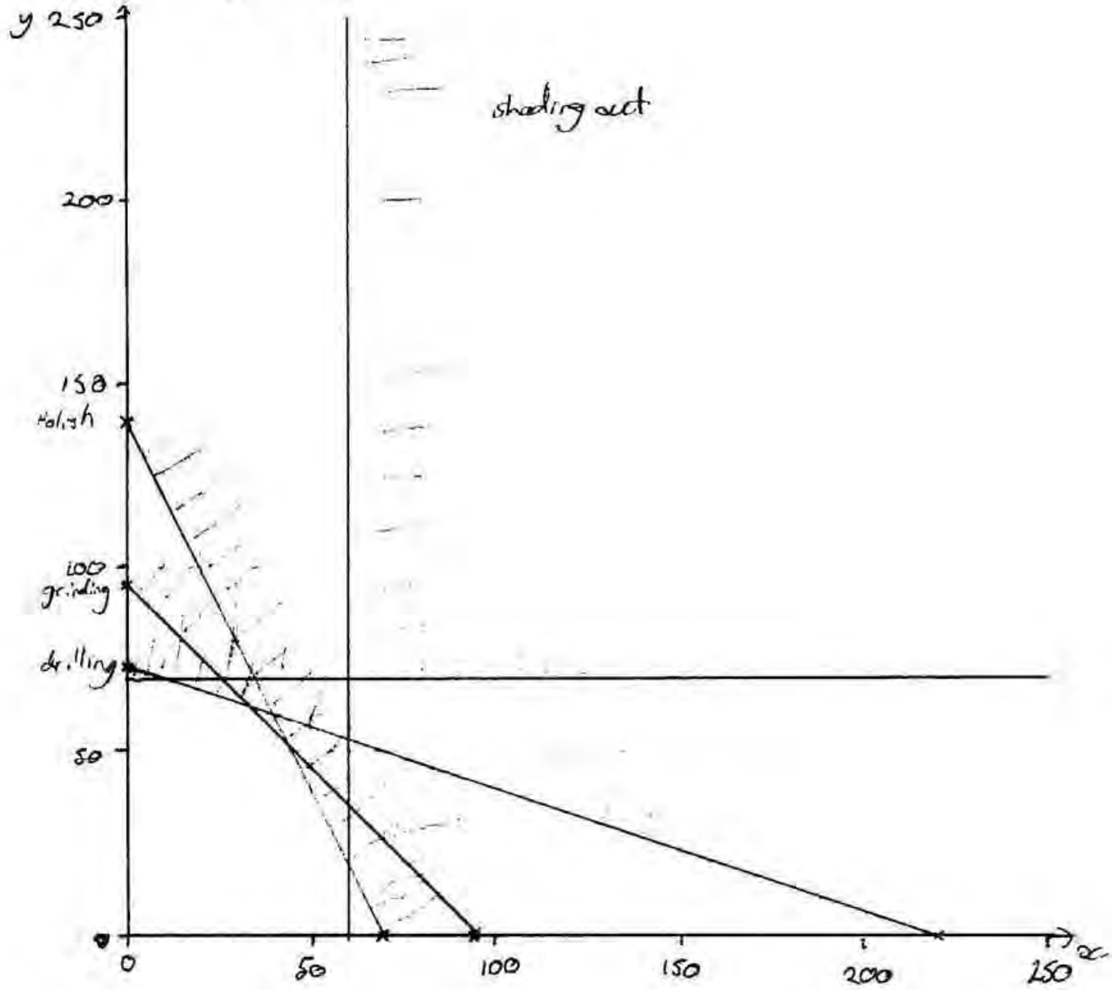
grinding $x + y \le 95$ $y \le -x + 95$

same for polishing $x + 0.5y \le 70$ $y \le -2x + 140$

3

$x \le 60$ $x \ge 0$

$y \le 70$ $y \ge 0$



Pmax new hours

$P_{max} = 300x + 600y$

5 extra drill	5 extra grind		
(10, 70)	(45, 50)	(60, 20)	(31, 63)
43000	43500	30000	47100
− (80×5)	− (150×5)		− (1150)
= $42600	$42750		= 45950

The increase in drilling and grinding hours is optimum for 31.5 rods and 63.5 pillars. Profit would increase by $950 as rods produced would change from 30 to 31 and pillars produced would change from 60 to 63.

4

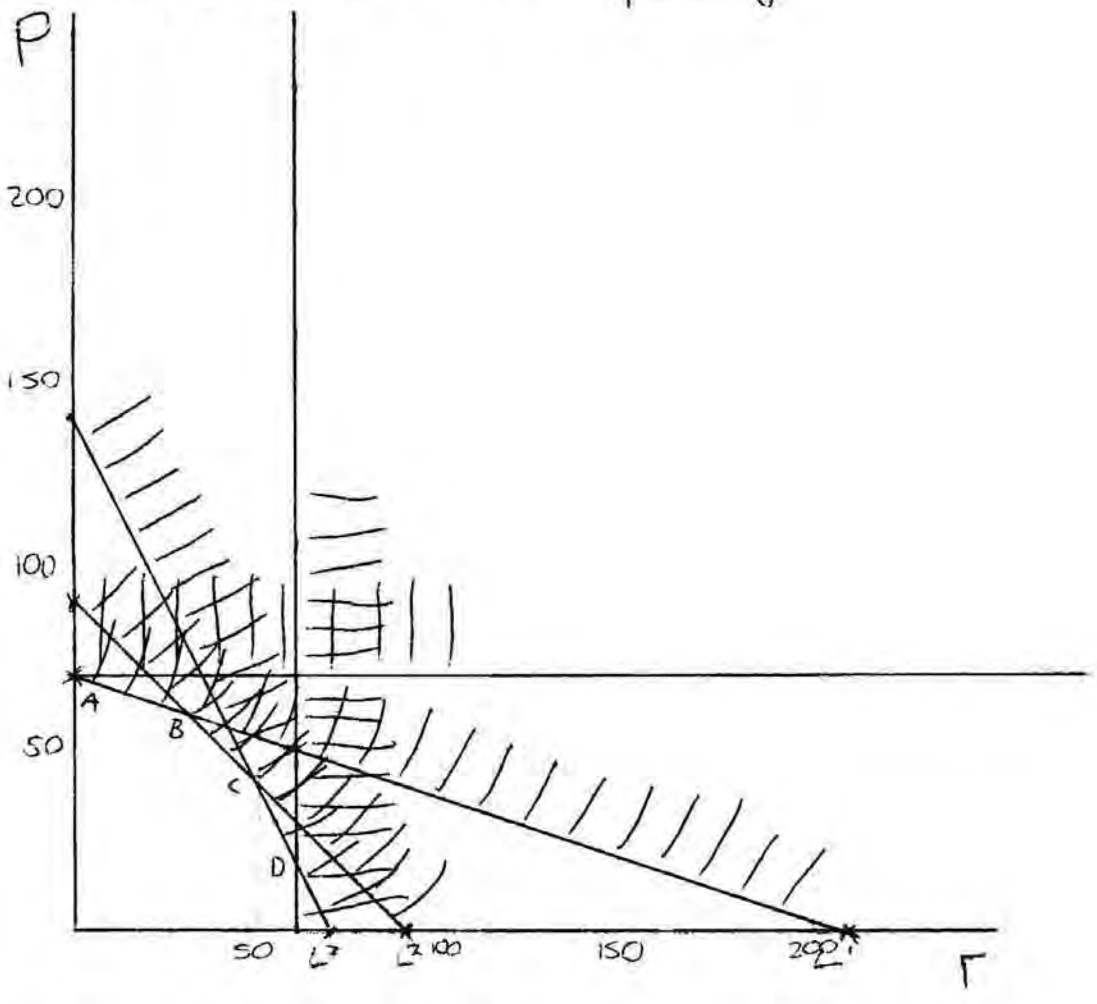
	Grade Boundary: High Merit
2.	For Merit the student is required to apply linear programming methods, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, and relating findings to a context or communicating thinking using appropriate mathematical statements. The student has shown evidence of relational thinking by identifying the feasible region for the system of linear inequalities **(1)** and identifying the number of rods and pillars to produce in order to maximise the profit **(2)**. The constraints for the increased hours have been provided **(3)**. The student has identified an optimal number of rods and pillars to produce for the increased hours **(4)**, but has not provided the evidence to support this claim. To be awarded Excellence the student would need to identify that only integral numbers of rods and pillars can be produced.

How many rods and pillars should Ace Machine Company produce to maximise profit without increasing hours?

$P = 300r + 600p$

$r \leq 60 \quad p \leq 70$

$0.5r + 1.5p \leq 105 \quad L^1$ (drilling)
$1r + 1p \leq 90 \quad L^2$ (grinding)
$1r + 0.5p \leq 70 \quad L^3$ (polishing)



1

A	B	C	D
70p + 0r	60p + 30r	40p + 50r	20p + 60r
300×0 + 600×70 =	300×30 + 600×60 =	300×50 + 600×40 =	300×600 + 600×20 =

A = \$42 000
B = \$45000
C = \$39 000
D = \$30 000

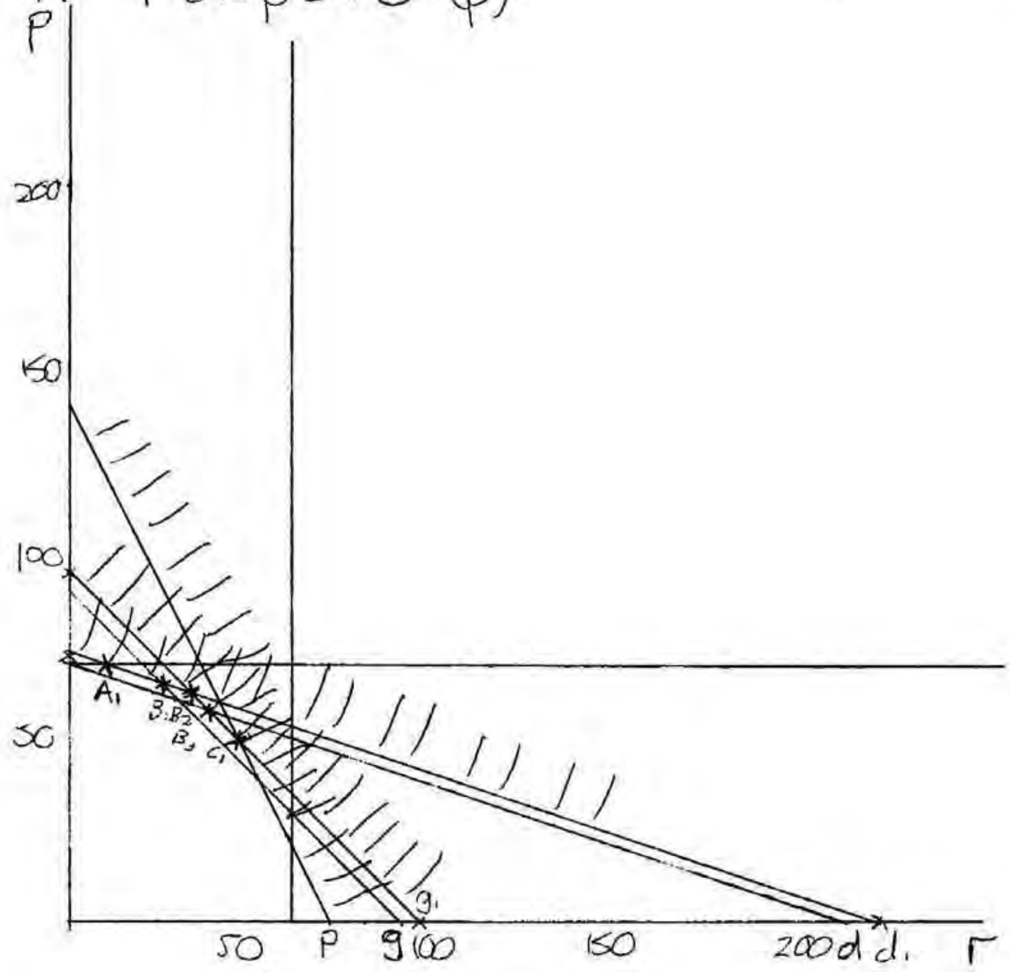
Ace Machine Company should produce 30 rods and 60 pillars to maximise profit at \$45 000 a week. 2

How would changes in drilling and grinding hours affect the optimum mix of these two machine parts?

$r \le 60$ $p \le 70$

$0.5r + 1.5p = 105$ (d) $0.5r + 1.5p = 110$ (d_1)
$1r + 1p = 90$ (g) $1r + 1p = 95$ (g_1)
$1r + 0.5p = 70$ (p)

3



A_1	B_1	B_2	B_3	C_1
300×10 +600×70 =45000 X	300×25 +600×65 =46500	300×32.5 +600×62.5 =46955	300×37.5 +600×57.5 =45750	300×45 +600×50 =43500 X

B_1 46500 − 400 = 46100 B_2 46955 − 1150 = 45805 X B_3 45750 − 45000

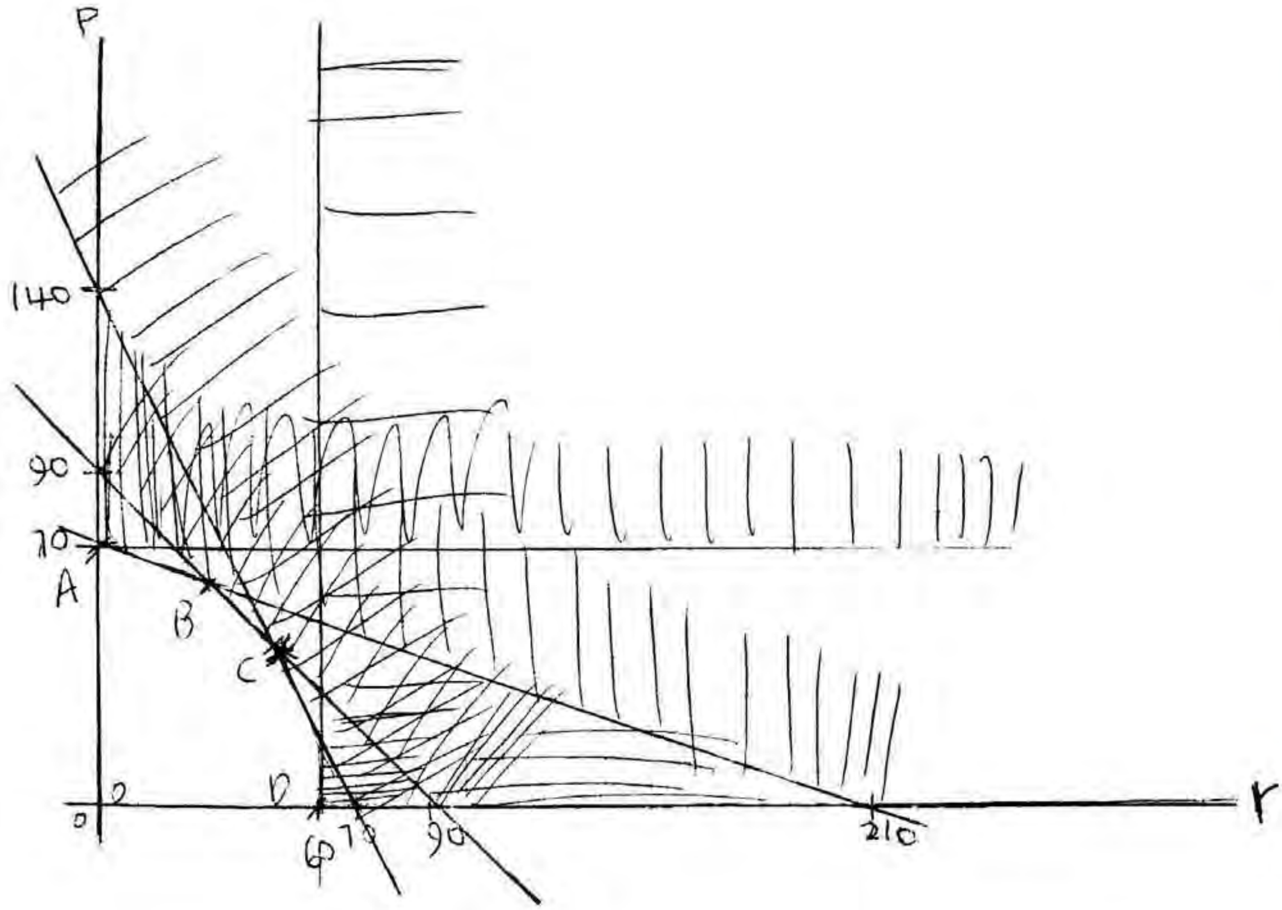
A change in drilling hours, will change the optimum mix to 25 rods and 65 pillars, at 46100 profit.

4

	Grade Boundary: Low Merit
3.	For Merit the student is required to apply linear programming methods, using relational thinking, in solving problems. This involves selecting and carrying out a logical sequence of steps, connecting different concepts and representations, demonstrating understanding of concepts, and relating findings to a context or communicating thinking using appropriate mathematical statements. The student has shown evidence of relational thinking by providing the feasible region for the system of linear inequalities **(1)** and identifying the number of rods and pillars to produce maximise the profit **(2)**. For a more secure Merit the student would need to communicate more accurately what each variable represents and consider how the changes in the hours would affect the optimal mix of machine parts.

Student 3: Low Merit

drilling $0.5r + 1.5p \leq 105$ (210, 70) $r \leq 60$
grinding $r + p \leq 90$ (90, 90) $p \leq 70$
$r + 0.5p \leq 70$ (70, 140)



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Vertex	co-ordinate	Profit ($300r + 600p$)
O	(0, 0)	0
A	(0, 70)	42000
B	(30, 60)	45000 ✓
C	(50, 40)	39000
D	(60, 0)	18000

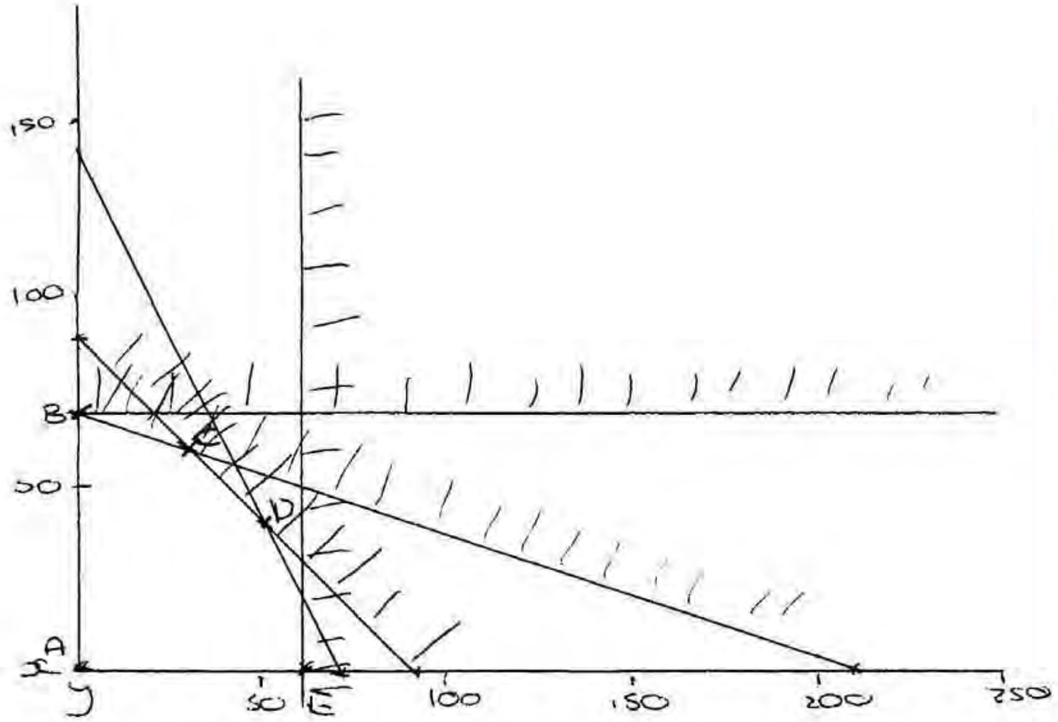
maximum profit is \$45000, 30 rods, 60 pillars. 2

	Grade Boundary: High Achieved
4.	For Achieved the student is required to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and also communicating using appropriate representations. The student has shown evidence of applying linear programming methods by forming the constraints **(1)** and using them to identify the feasible region **(2)**. The student has also evaluated the profit for each vertex of the feasible region **(3)**. To be awarded Merit the student would need to identify the vertex which maximises the profit function to make a recommendation regarding the number rods and pillars to produce.

Student 4: High Achieved

Constraints	r = x p = y	intercept x, y
$x \leq 60$		60, 0
$y \leq 70$		0, 70
$105 \geq 0.5x + 1.5y$		210, 70
$90 \geq x + y$		90, 90
$70 \geq x + 0.5y$		70, 140

1



2

	x, y	
A	0, 0	0
B	0, 70	42,000
C	30, 60	45,000
D	50, 40	39,000
E	60, 0	18,000

3

	Grade Boundary: Low Achieved
5.	For Achieved the student is required to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and also communicating using appropriate representations. The student has shown evidence of linear programming methods by providing the equations of the inequalities for the drilling, grinding and polishing **(1)**, and has also provided the feasible region identified by the five constraints **(2)**. For a more secure Achieved the student would need to indicate what is represented by each variable.

Student 5: Low Achieved

graph without increased hours

constrants

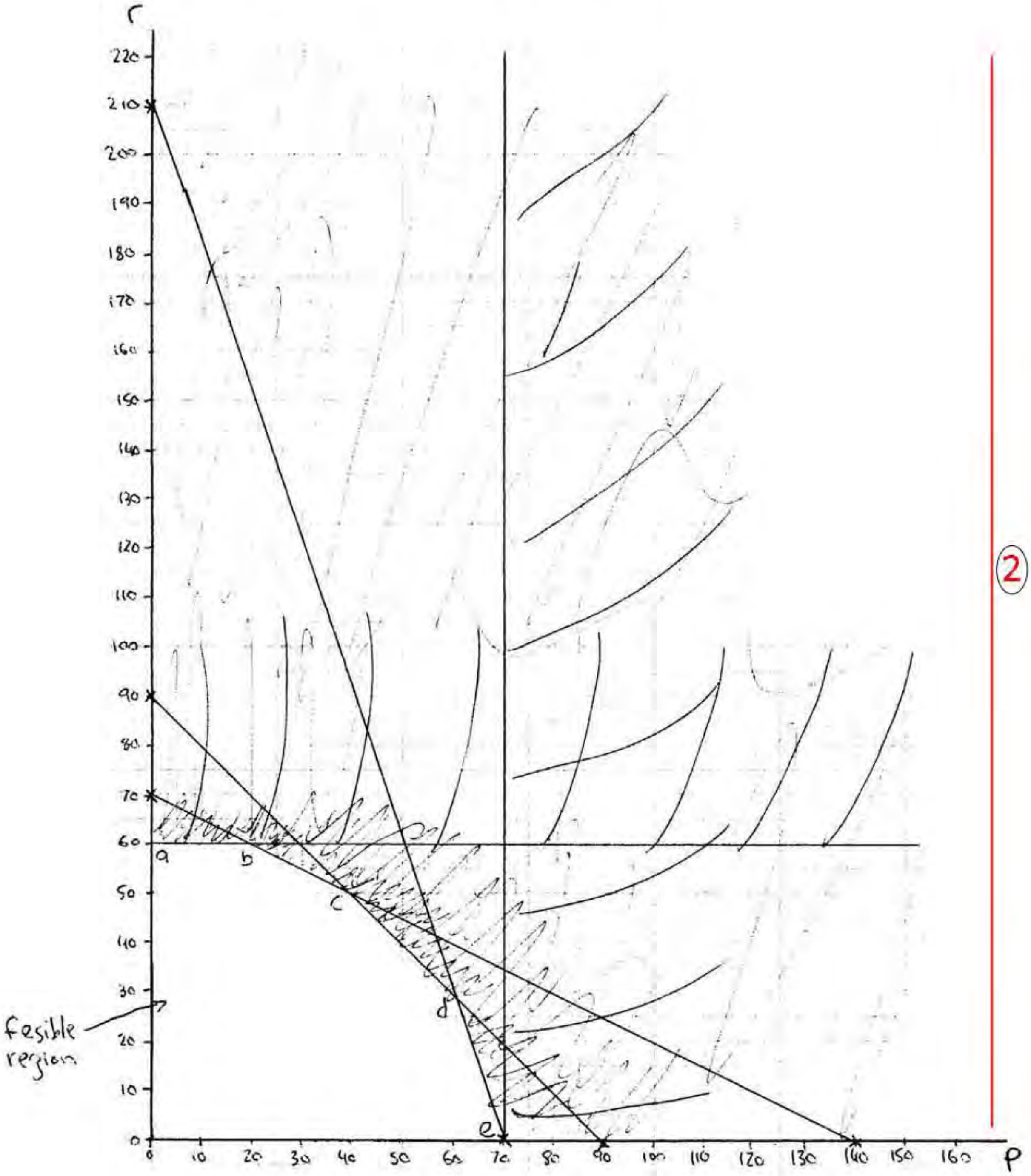
$r \leq 60$ $p \leq 70$

drilling $0.5r + 1.5p \leq 105$

grinding $1r + 1p \leq 90$

polishing $1r + 0.5p \leq 70$

1



2

a = (0,60) b = (20,60) c = (40,50) d = (60,30) e = (70,0)

	(0,60)	(20,60)	(40,50)	(60,30)	(70,0)
P = 300r + 600p					

	Grade Boundary: High Not Achieved
6.	For Achieved the student is required to apply linear programming methods in solving problems. This involves selecting and using methods, demonstrating knowledge of concepts and terms and also communicating using appropriate representations. The student has provided the equations of all the inequalities **(1)**. To be awarded Achieved the student would need to correctly graph the regions relating to the constraints for drilling or polishing.

Student 6: High Not Achieved

$P = 300x + 600y$

x = rods $\quad y$ = pillars

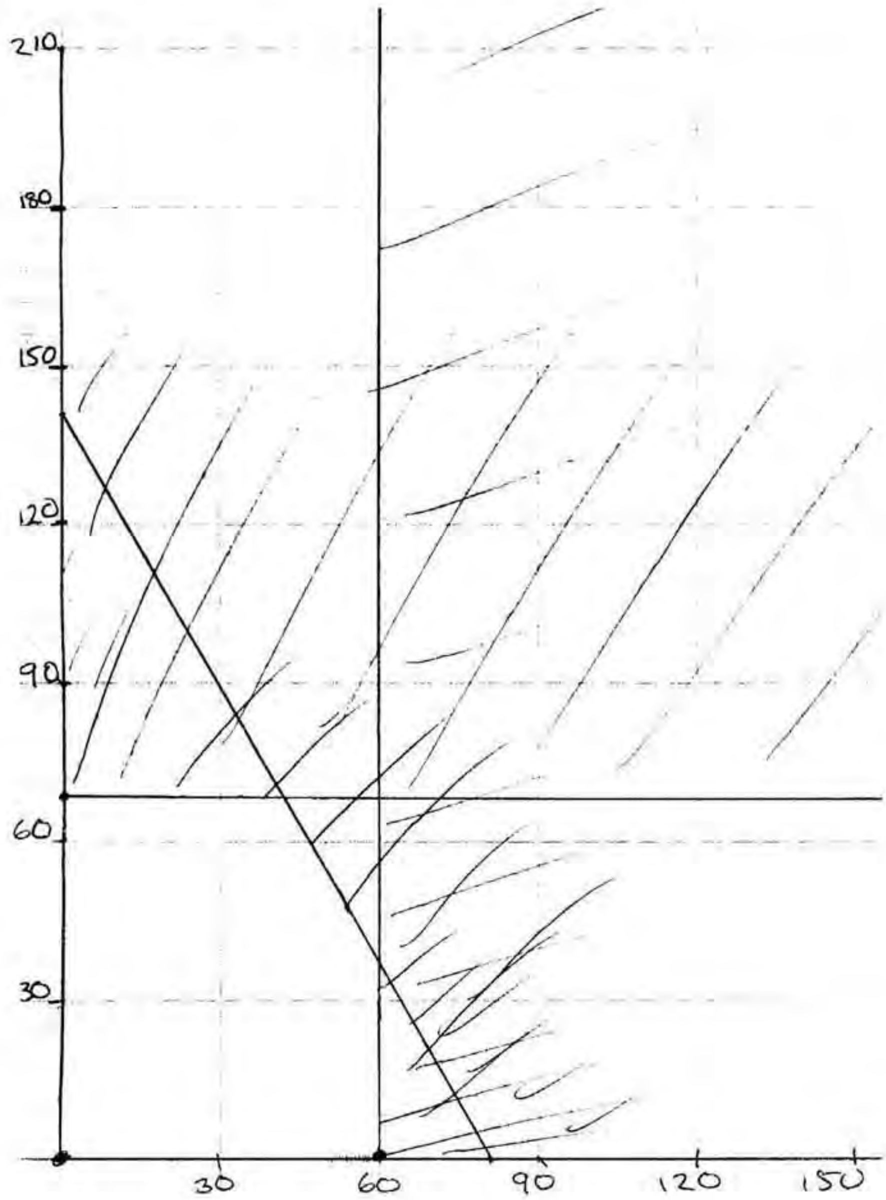
$x \leq 60 \quad y \leq 70$

$0.5x + 1.5y \leq 105$ (D)

$x + y \leq 90$ (G)

$x + 0.5y \leq 70$ (P)

1



	Profit
(0, 0)	
(0, 70)	
(40, 70)	
(60, 35)	
(60, 0)	